Autonomous Energy Grid
optimization

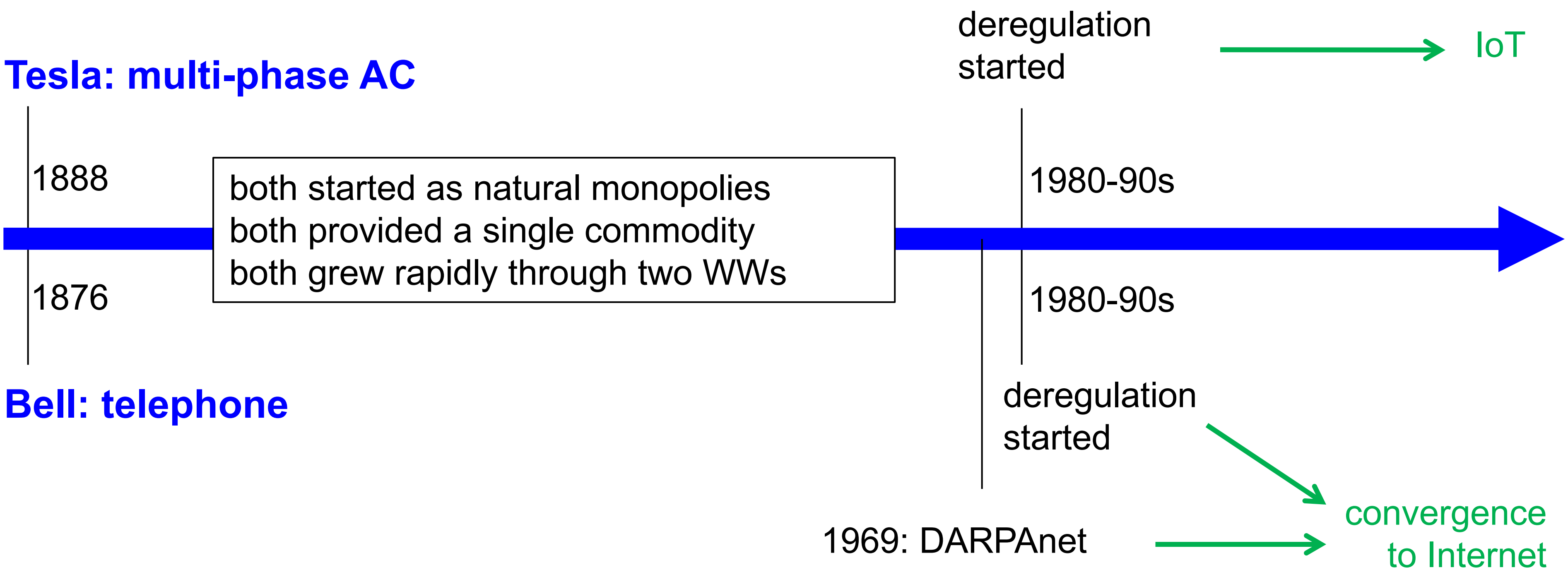
Steven Low

Caltech

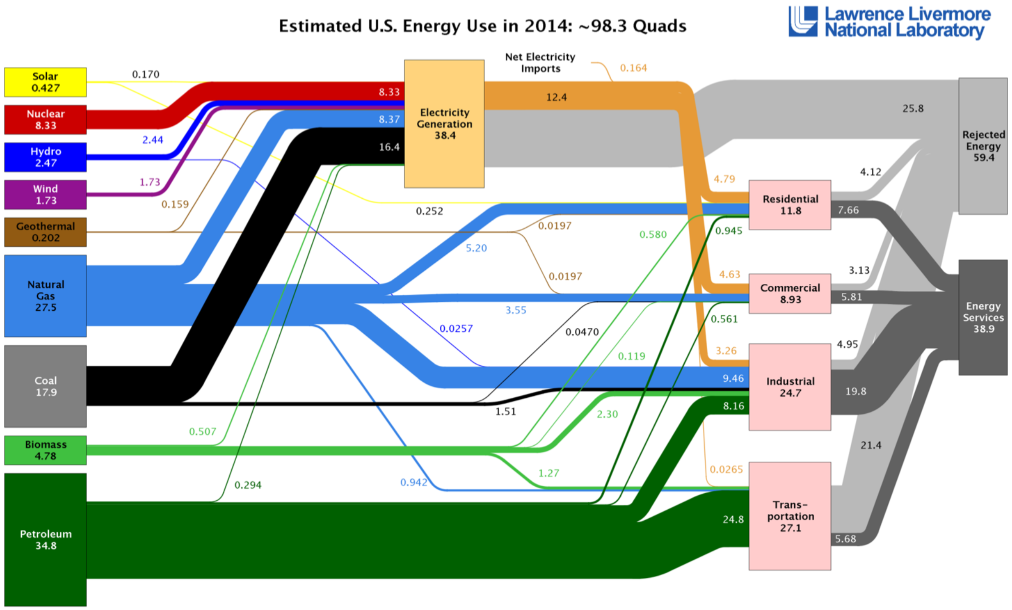
April 2018

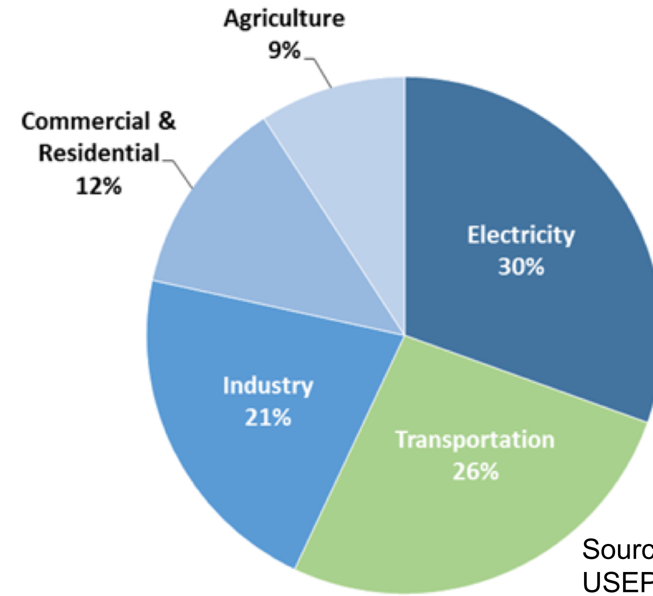
Watershed moment

Energy network will undergo similar architectural transformation that phone network went through in the last two decades to become the world's largest and most complex IoT



Electricity gen & transportation





Source: USEPA

They consume the most energy

- Consume 2/3 of all energy in US (2014)

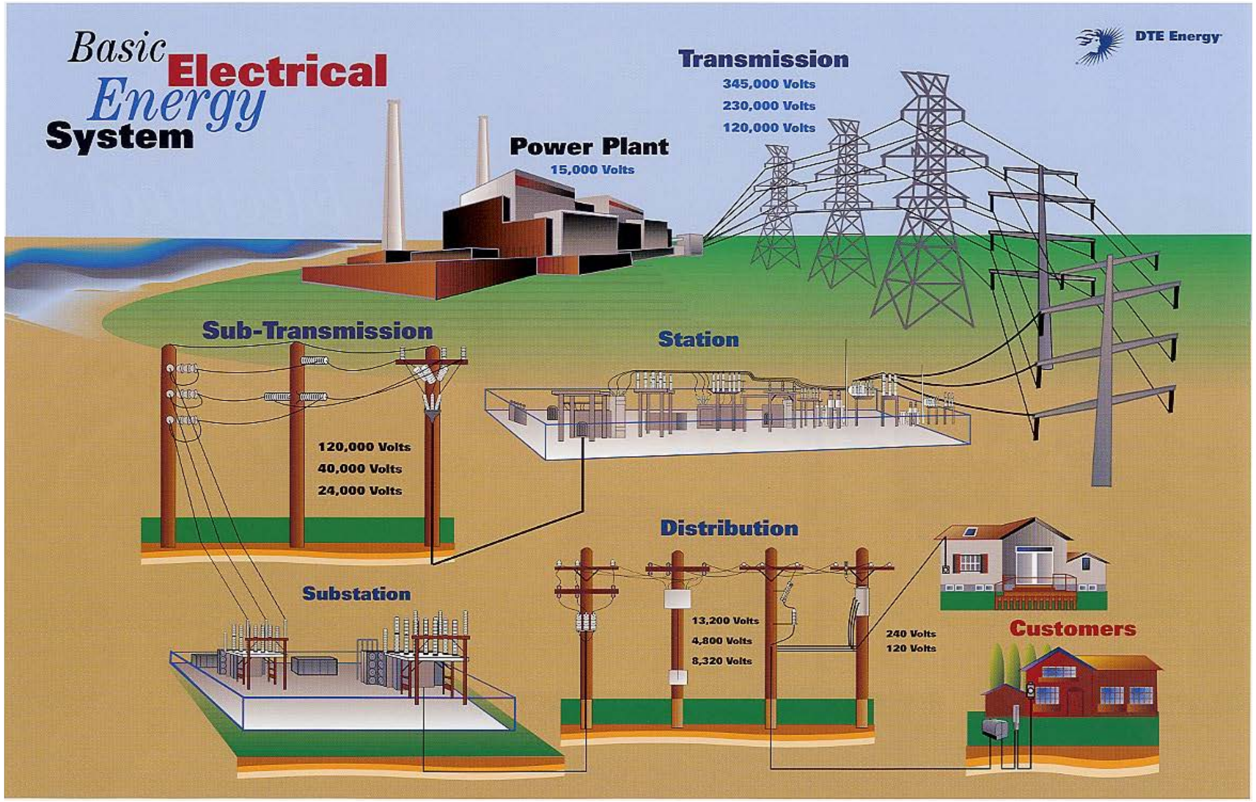
They emit the most greenhouse gases

- Emit >1/2 of all greenhouse gases in US (2014)

To drastically reduce greenhouse gases

- Generate electricity from renewable sources
- Electrify transportation

Today's grid



Few large generators

- ~10K bulk generators, actively controlled

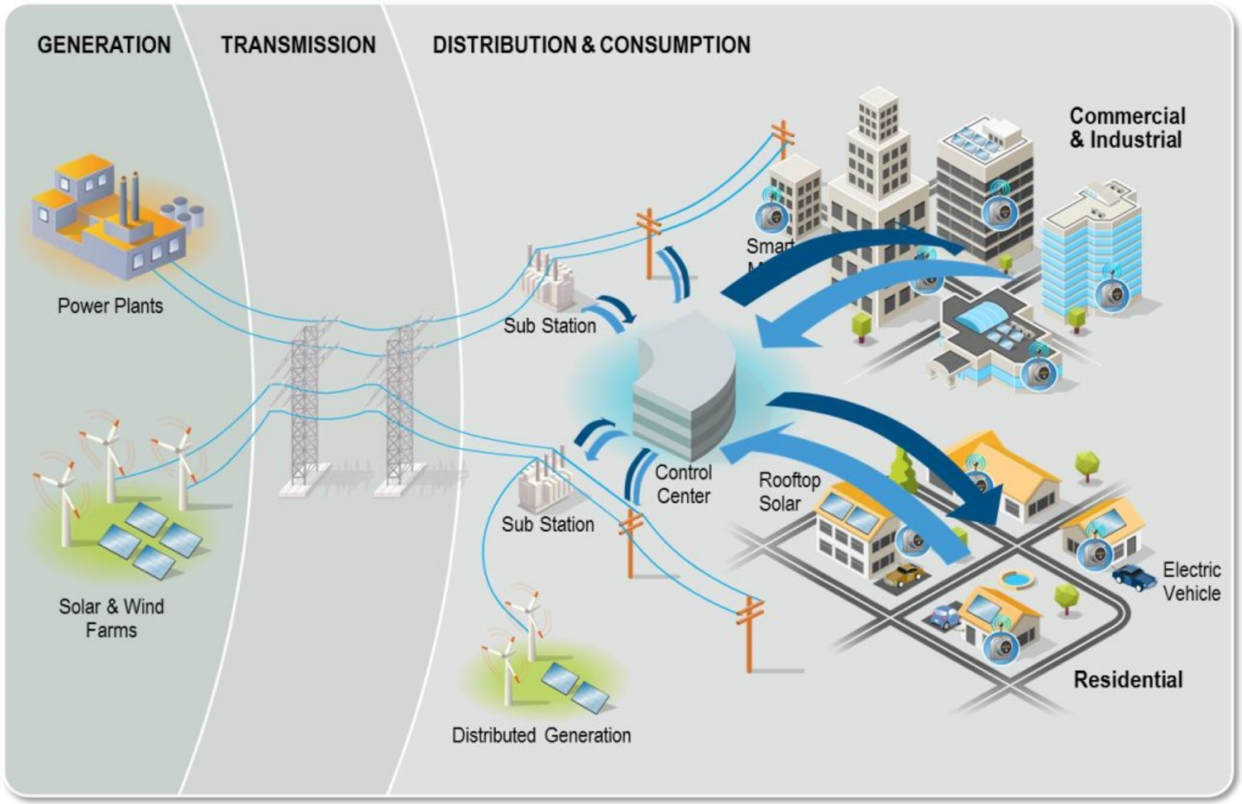
Many dump loads

- 131M customers, ~billion passive loads

Control paradigm: schedule supply to match demand

- Centralized, human-in-the-loop, worst case, deterministic

Future grid



Wind and solar farms are not dispatchable

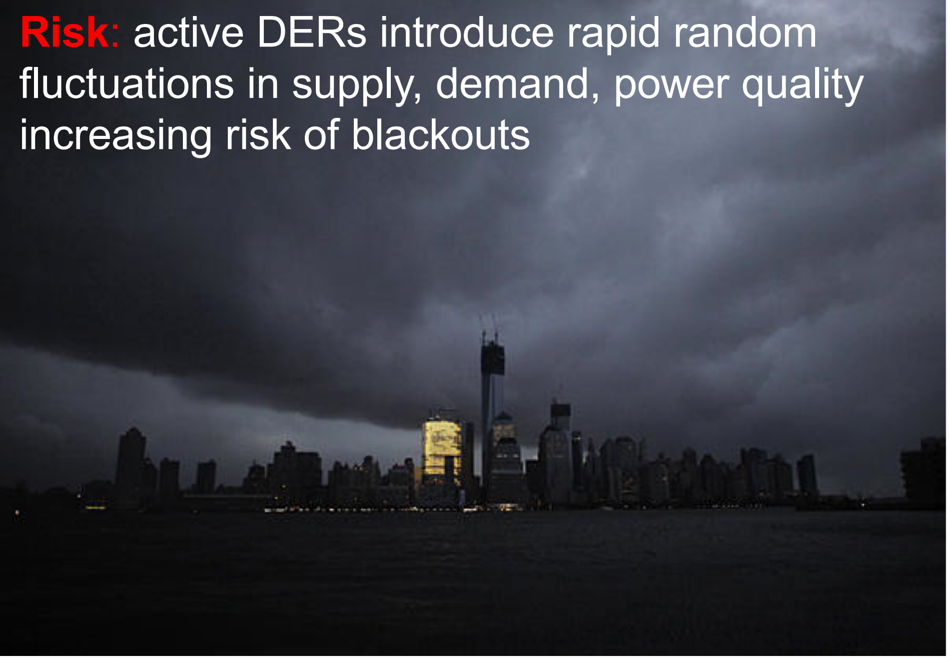
- Many small distributed generations

Network of distributed energy resources (DERs)

- EVs, smart buildings/appliances/inverters, wind turbines, storage

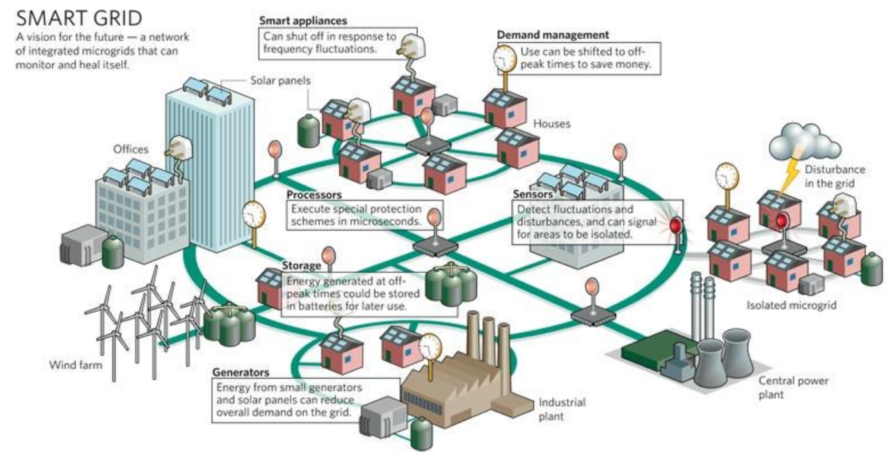
Control paradigm: match demand to volatile supply

- Distributed, real-time feedback, risk limiting, robust

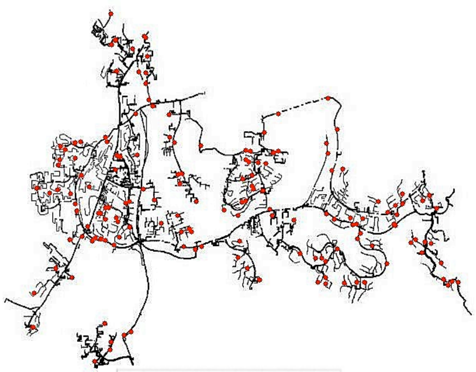


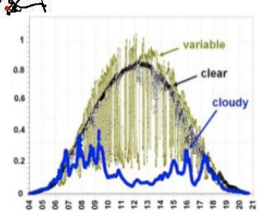
Risk: active DERs introduce rapid random fluctuations in supply, demand, power quality increasing risk of blackouts

Opportunity: active DERs enables realtime dynamic network-wide feedback control, improving robustness, security, efficiency



Caltech research: distributed control of networked DERs





- Foundational theory, practical algorithms, concrete applications
- Integrate engineering and economics
- Active collaboration with industry

NSF · arpa-e · SOUTHERN CALIFORNIA EDISON · RESNICK INSTITUTE science + energy + sustainability · Skoltech Skolkovo Institute of Science and Technology

Autonomous energy grid

Computational challenge

- nonlinear models, nonconvex optimization

Scalability challenge

- billions of intelligent DERs

Increased volatility

- in supply, demand, voltage, frequency

Limited sensing and control

- design of/constraint from cyber topology

Incomplete or unreliable data

- local state estimation & system identification

Data-driven modeling and control

- real-time at scale

many other important problems, inc. economic, regulatory, social, ...

Ben Kroposki, 2007 https://www.nrel.gov/grid/autonomous-energy.html

Outline

Relaxations of AC OPF

- Dealing with nonconvexity

Realtime AC OPF

- Dealing with volatility

Optimal placement

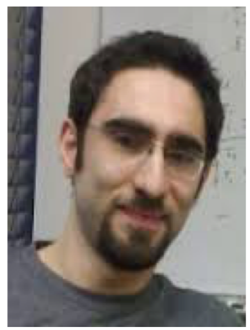
- Dealing with limited sensing/control

Relaxations of AC OPF

dealing with nonconvexity


Bose (UIUC)


Chandy


Farivar (Google)


Gan (FB)


Lavaei (UCB)


Li (Harvard)

many others at & outside Caltech …

Low, Convex relaxation of OPF, 2014
http://netlab.caltech.edu

Optimal power flow (OPF)

OPF is solved routinely for

- network control & optimization decisions
- market operations & pricing
- at timescales of mins, hours, days, …

Non-convex and hard to solve

- Huge literature since 1962
- Common practice: DC power flow (LP)
- Also: Newton-Raphson, interior point, …

$$\min\ c(x) \quad \text{s.t.} \quad F(x) = 0,\ x \le \overline{x}$$

Optimal power flow

min	$\mathrm{tr}\left(CVV^H\right)$	gen cost, power loss
over	(V, s, l)	
subject to	$s_j = \mathrm{tr}\left(Y_j^H VV^H\right)$	power flow equation
	$l_{jk} = \mathrm{tr}\left(B_{jk}^H VV^H\right)$	line flow
	$\underline{s}_j \le s_j \le \overline{s}_j$	injection limits
	$\underline{l}_{jk} \le l_{jk} \le \overline{l}_{jk}$	line limits
	$\underline{V}_j \le \lvert V_j \rvert \le \overline{V}_j$	voltage limits

- Y_j^H describes network topology and impedances
- s_j is net power injection (generation) at node j

Optimal power flow

min	$\operatorname{tr}\left(CVV^H\right)$	gen cost, power loss
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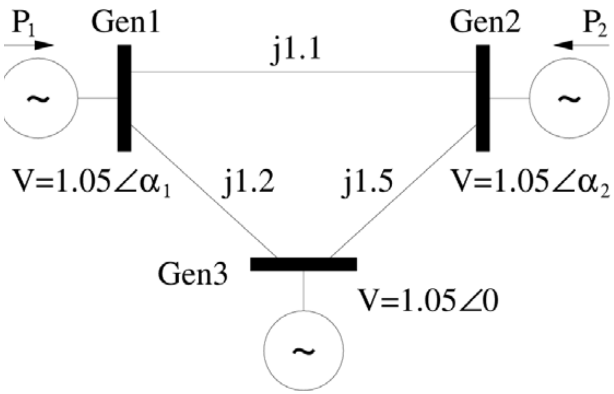
nonconvex feasible set (nonconvex QCQP)

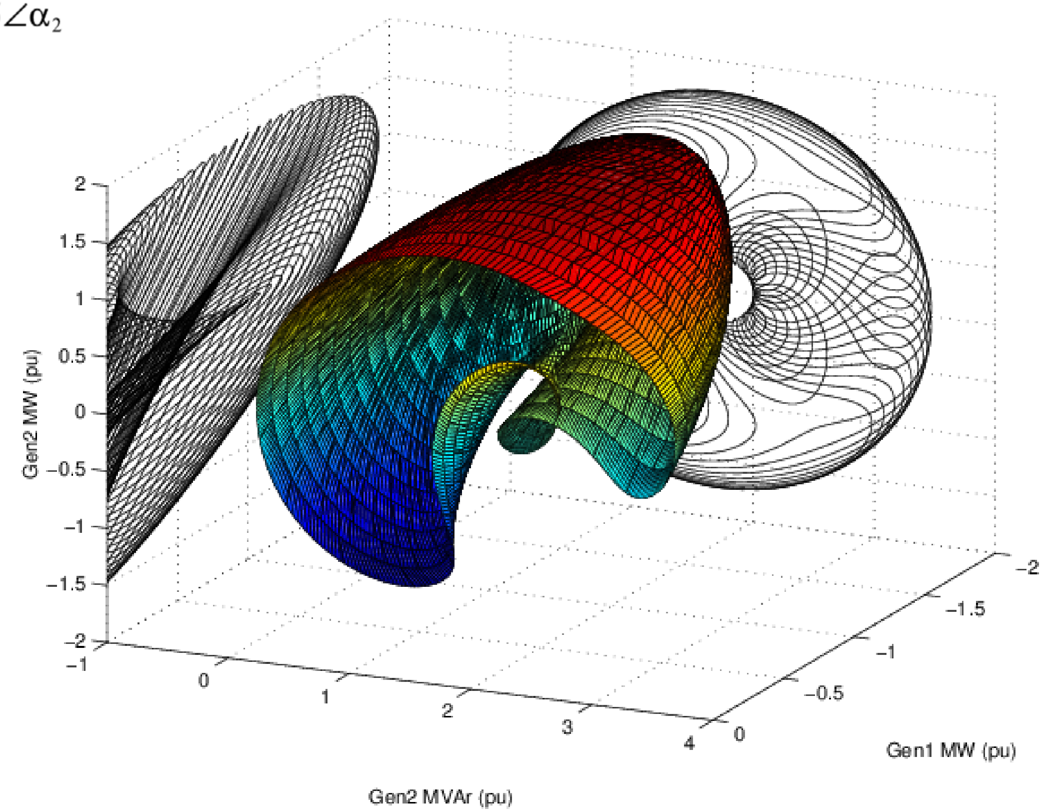
- Y_j^H not Hermitian (nor positive semidefinite)
- C is positive semidefinite (and Hermitian)

Optimal power flow

OPF problem underlies numerous applications

- nonlinearity of power flow equations ➔ nonconvexity





Ian Hiskens, Michigan

Dealing with nonconvexity

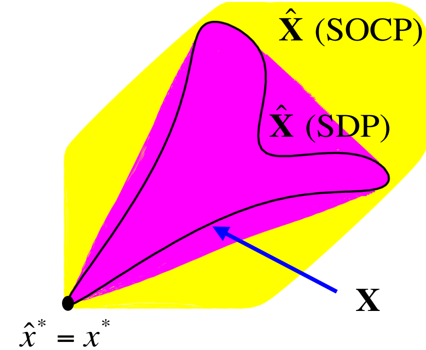
Linearization

- DC approximation

Convex relaxations

- Semidefinite relaxation (Lasserre hierarchy)
- QC relaxation (van Hentenryck)
- Strong SOCP (Sun)

Equivalent feasible sets



$$\min \quad \operatorname{tr} CVV^H$$

$$\text{subject to} \quad \underline{s}_j \le \operatorname{tr}\left(Y_j^H V V^H\right) \le \bar{s}_j \qquad \underline{v}_j \le |V_j|^2 \le \bar{v}_j$$

quadratic in V
linear in W

Equivalent problem:

$$\min \quad \operatorname{tr} CW$$

$$\text{subject to} \quad \underline{s}_j \le \operatorname{tr}\left(Y_j^H W\right) \le \bar{s}_j \qquad \underline{v}_j \le W_{jj} \le \bar{v}_j$$

$$W \ge 0, \quad \operatorname{rank} W = 1$$

convex in W
except this constraint

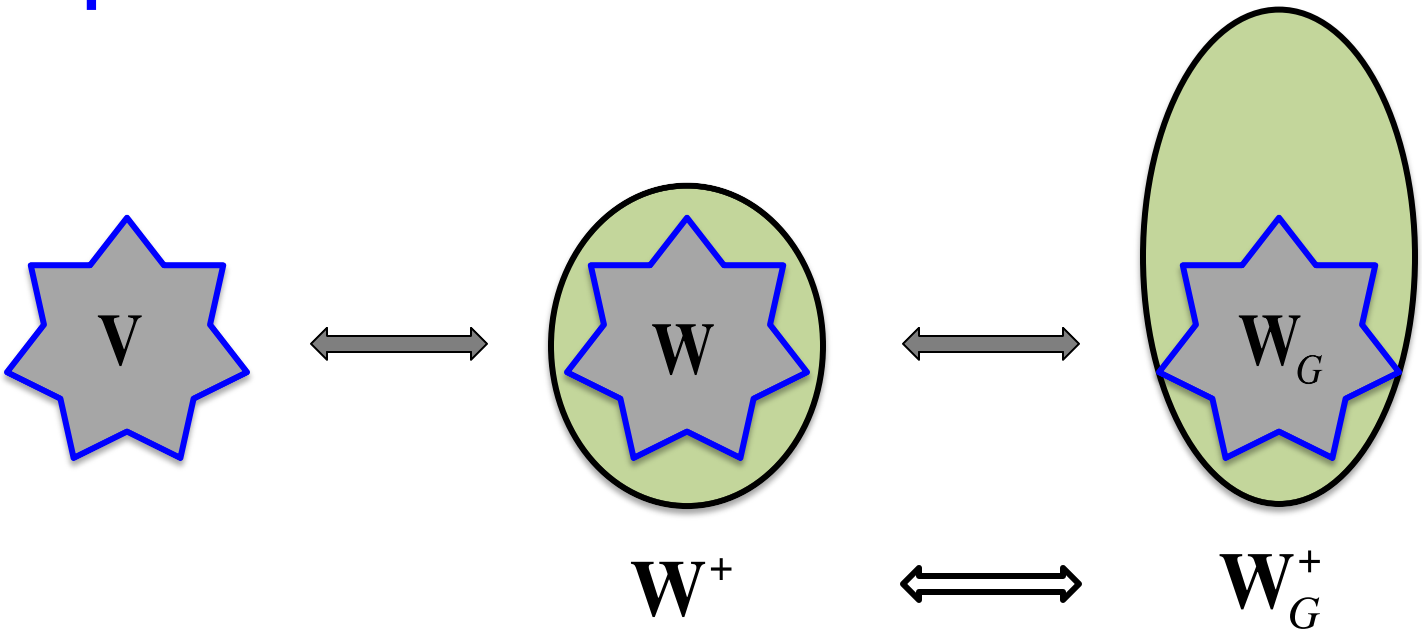
Solution strategy

OPF: $$\min_{x \in \mathbf{X}} f(x)$$

relaxation: $$\min_{\hat{x} \in \mathbf{X}^+} f(\hat{x})$$

If optimal solution $\hat{x}^*$ satisfies easily checkable conditions, then optimal solution x^* of OPF can be recovered

Equivalent relaxations



Theorem

- Radial G: SOCP is equivalent to SDP ($\mathbf{V} \subseteq \mathbf{W}^+ \cong \mathbf{W}_G^+$)
- Mesh G: SOCP is strictly coarser than SDP

For radial networks: always solve SOCP !

Exact relaxation

For radial networks, sufficient conditions on

- power injections bounds, or
- voltage upper bounds, or
- phase angle bounds

Exact relaxation

QCQP (C, C_k)

$$\begin{array}{lll} \min & \mathrm{tr}\left(C x x^H\right) & \\ \text{over} & x \in \mathbf{C}^n & \\ \text{s.t.} & \mathrm{tr}\left(C_k x x^H\right) \le b_k & k \in K \end{array}$$

graph of QCQP

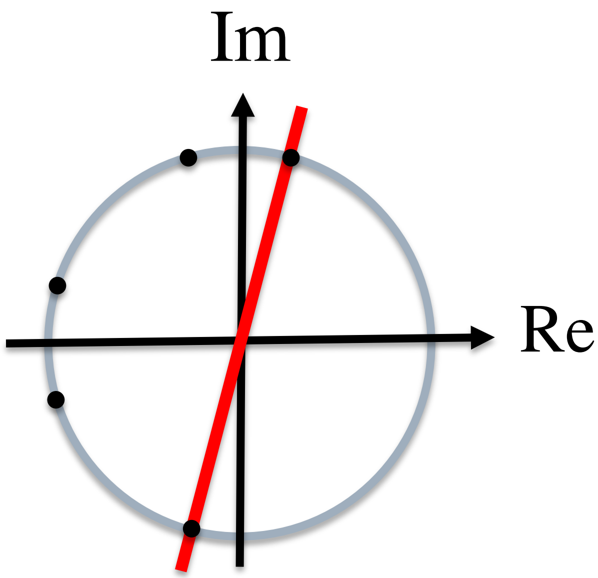
$G\left(C, C_k\right)$ has edge (i, j) $\Leftrightarrow$

$C_{ij} \neq 0$ or $\left[C_k\right]_{ij} \neq 0$ for some k

QCQP over tree

$G\left(C, C_k\right)$ is a tree

Exact relaxation



QCQP (C, C_k)

$$\begin{array}{lll} \min & \mathrm{tr}\left(C x x^H\right) & \\ \text{over} & x \in \mathbf{C}^n & \\ \text{s.t.} & \mathrm{tr}\left(C_k x x^H\right) \le b_k & k \in K \end{array}$$

Key condition

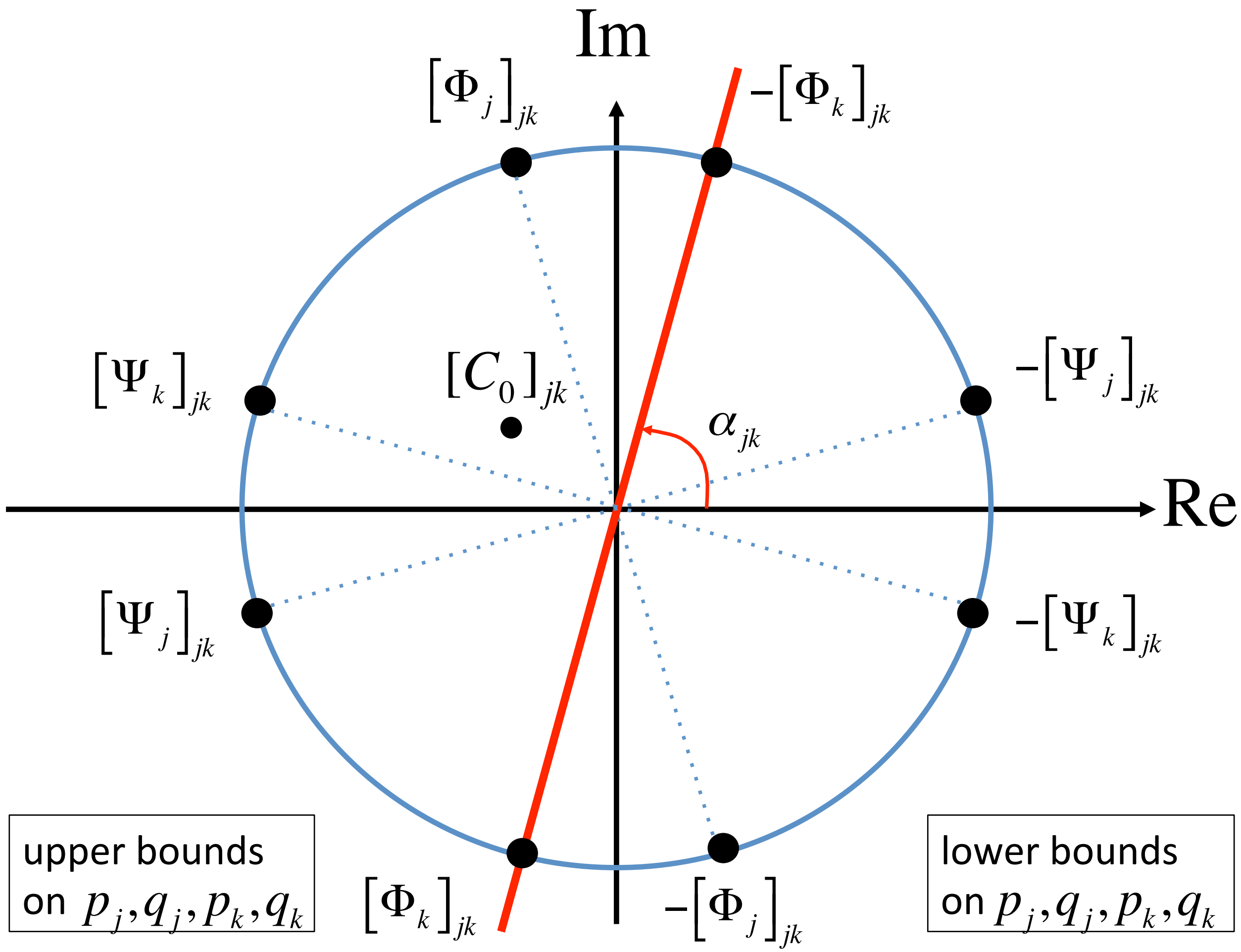
$i \sim j:\ \left(C_{ij}, [C_k]_{ij},\ \forall k\right)$ lie on half-plane through 0

Theorem

SOCP relaxation is exact for QCQP over tree

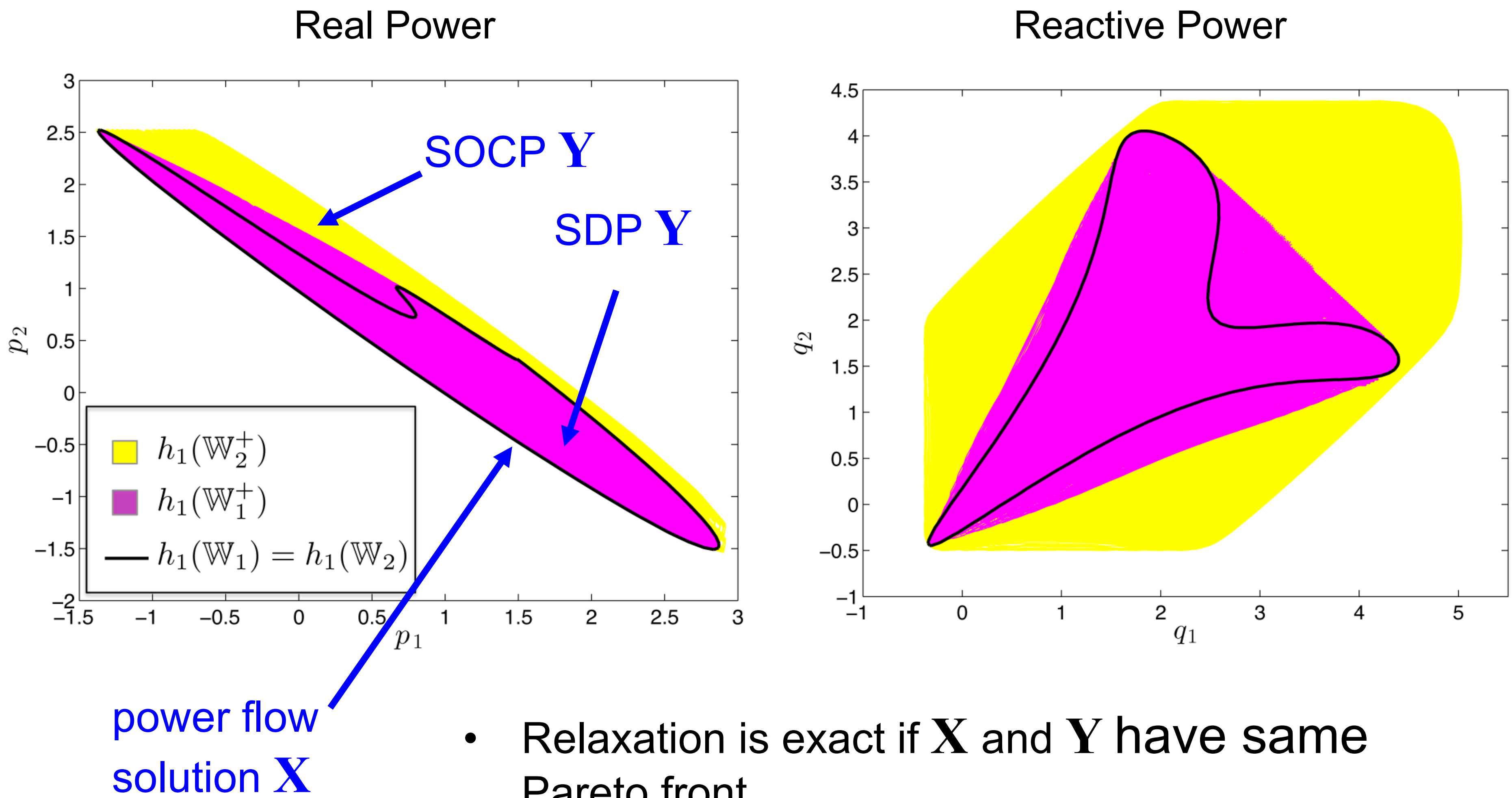
Bose et al 2012, 2014
Sojoudi, Lavaei 2013

Implication on OPF



Not both lower & upper bounds on real & reactive powers at both ends of a line can be finite

Example



- Relaxation is exact if $\mathbf{X}$ and $\mathbf{Y}$ have same Pareto front
- SOCP is faster but coarser than SDP

Bose, Low, Teeraratkul, Hassibi TAC 2015

Potential benefits

IEEE test systems		SDP cost	MATPOWER cost
Syst.	$\text{rank}(\overline{X}_0)$	J°	$\overline{J}$
9	1	5296.7	5296.7
30	1	576.9	576.9
118	1	129661	129661
14A	1	**8092.8**	**9093.8**

12.4% lower cost than solution from nonlinear solver MATPOWER

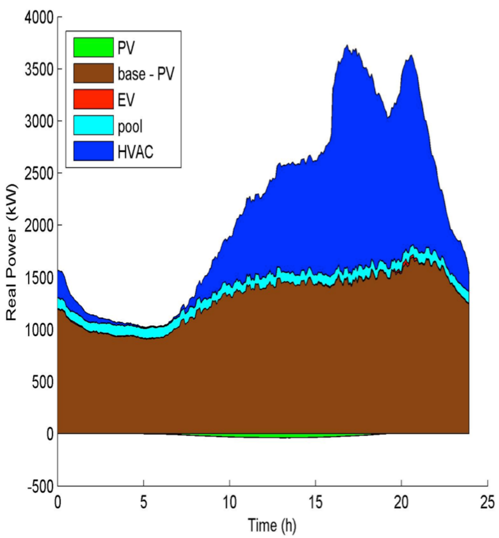
[Louca, Seiler, Bitar 2013]

Potential benefits

Case study on an SCE feeder

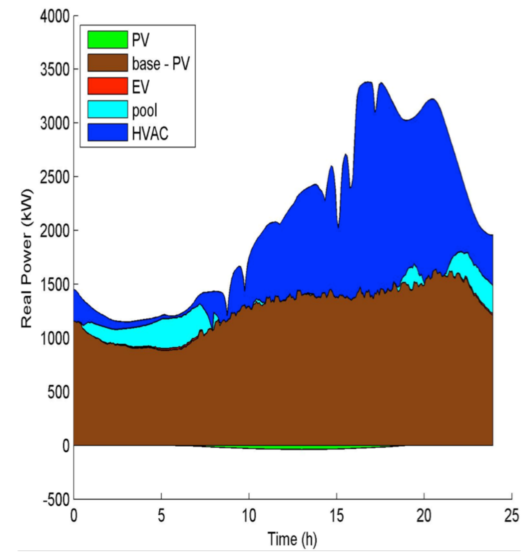
- Southern California
- 1,400 residential houses, ~200 commercial buildings
- Controllable loads: EV, pool pumps, HVAC, PV inverters
- Formulated as an OPF problem, multiphase unbalanced radial network

baseline



peak load reduction: 8%
energy cost reduction: 4%

optimized



Realtime AC OPF

for tracking

Gan (FB)

Tang (Caltech)

Dvijotham (DeepMind)

See also: Dall'Anese et al, Bernstein et al, Hug & Dorfler et al, Callaway et al

Gan & L, JSAC 2016
Tang et al, TSG 2017

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x, \; y$$

$$\text{s. t.}$$

controllable devices (↑ x)

uncontrollable state (↑ y)

OPF

$$\begin{aligned} \min \quad & c_0(y) + c(x) \\ \text{over} \quad & x,\ y \\ \text{s.t.} \quad & F(x,y) = 0 \end{aligned}$$

power flow equations

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x,\ y$$

$$\text{s. t.} \quad F(x,y) = 0$$

power flow equations

$$y \le \overline{y}$$

operational constraints

$$x \in X := \left\{ \underline{x} \le x \le \overline{x} \right\}$$

capacity limits

$$\text{Assume: } \frac{\partial F}{\partial y} \ne 0 \quad \Rightarrow \quad y(x) \quad \text{over } X$$

OPF

$$\min_x \quad c_0(y(x)) + c(x)$$

$$\text{s.t.} \quad y(x) \le \overline{y}$$

$$x \in X := \{\underline{x} \le x \le \overline{x}\}$$

Theorem [Huang, Wu, Wang, & Zhao. TPS 2016]
For DistFlow model, controllable (feasible) region

$$\{x \mid y(x) \le \overline{y}, x \in X\}$$

is convex (despite nonlinearity of $y(x)$)

Static OPF

$$\begin{array}{ll} \min & f(x, y(x);\ \mu) \\ \text{over} & x \in X \end{array}$$

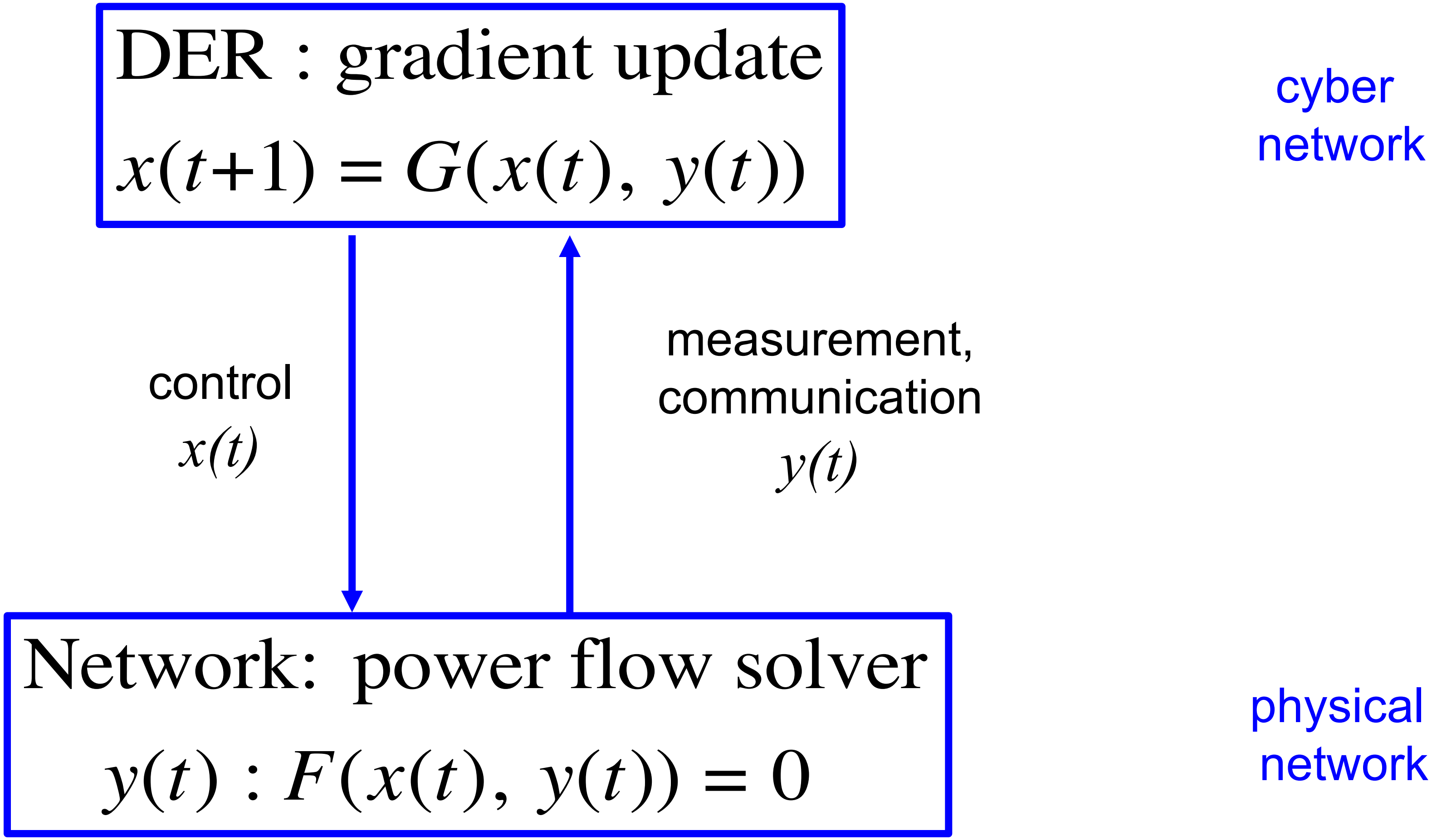
gradient projection algorithm:

$$x(t+1) = \left[x(t) - \eta \frac{\partial f}{\partial x}(t) \right]_X \qquad \text{active control}$$

$$y(t) = y(x(t)) \qquad \text{law of physics}$$

[Gan & Low, JSAC 2016]

Online (feedback) perspective



- Explicitly exploits network as power flow solver
- Naturally tracks changing network conditions

Drifting OPF

$$\min_x \quad c_0(y(x)) + c(x)$$
$$\text{s.t.} \quad y(x) \le \overline{y}$$
$$x \in X$$

static OPF

$$\min_x \quad c_0(y(x),\gamma_t) + c(x,\gamma_t)$$
$$\text{s.t.} \quad y(x,\gamma_t) \le \overline{y}$$
$$x \in X$$

drifting OPF

Drifting OPF

$$\begin{array}{ll} \min & f_t(x, y(x);\ \mu_t) \\ \text{over} & x \in X_t \end{array}$$

Quasi-Newton algorithm:

$$x(t+1) = \left[x(t) - \eta \left(H(t) \right)^{-1} \frac{\partial f_t}{\partial x}(x(t)) \right]_{X_t} \qquad \text{active control}$$

$$y(t) = y(x(t)) \qquad \text{law of physics}$$

[Tang, Dj & Low, 2017]

Tracking performance

$$\text{error} := \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

control error

Tracking performance

$$\text{error} := \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

Theorem

$$\text{error} \leq \frac{\varepsilon}{\sqrt{\lambda_M / \lambda_m} - \varepsilon} \cdot \frac{1}{T}\sum_{t=1}^{T}\left(\left\| x^{*}(t) - x^{*}(t-1) \right\| + \Delta_t \right) + \delta$$

avg rate of drifting

- of optimal solution
- of feasible injections

Tracking performance

$$\text{error} := \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

Theorem

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error in Hessian approx

Tracking performance

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Theorem

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"condition number" of Hessian

Tracking performance

$$\text{error} := \frac{1}{T}\sum_{t=1}^{T}\left\| x^{\text{online}}(t) - x^{*}(t) \right\|$$

Theorem

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"initial distance" from $x^{*}(t)$

Implementation

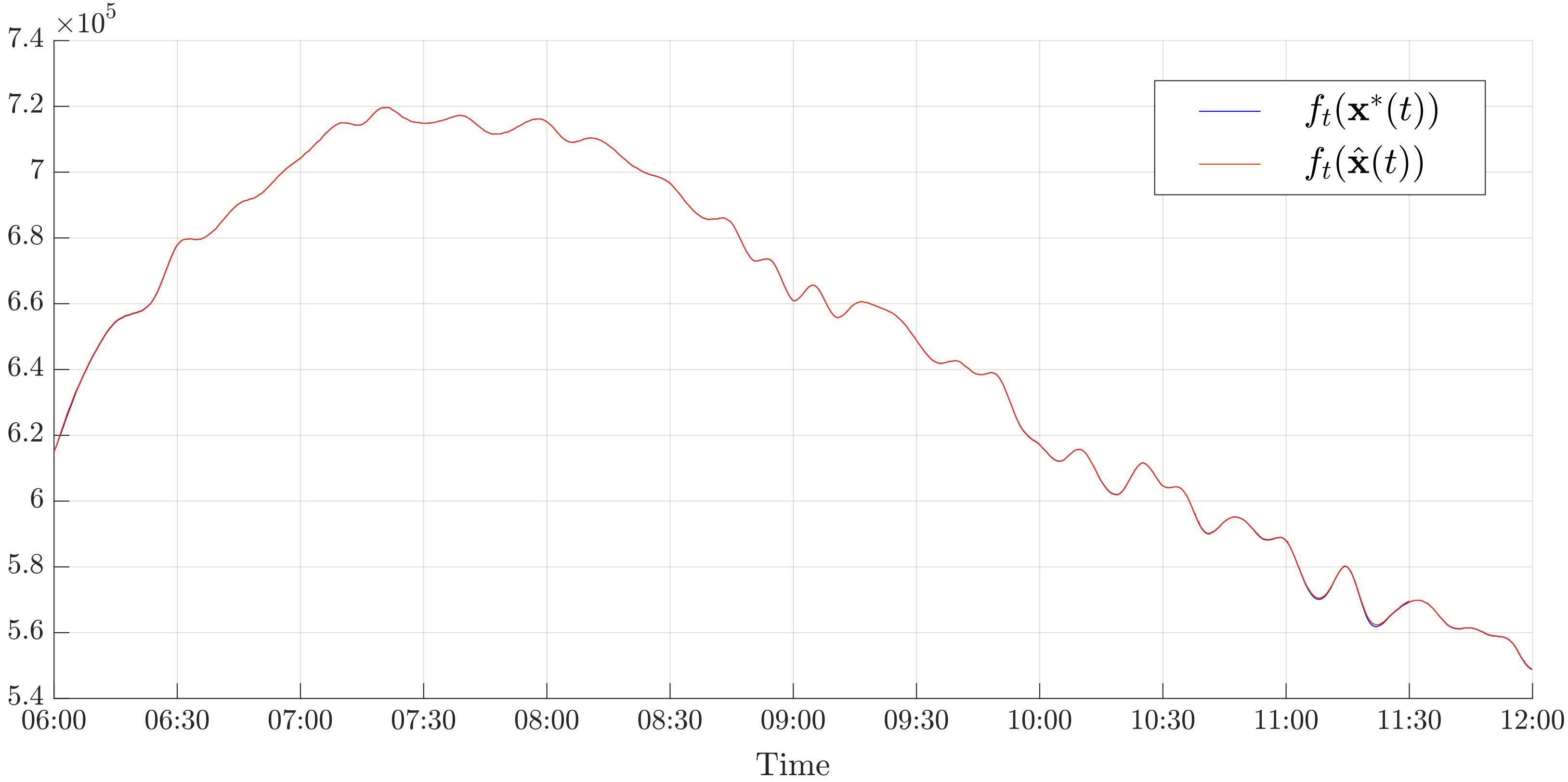
Implement L-BFGS-B

- More scalable
- Handles (box) constraints X

Simulations

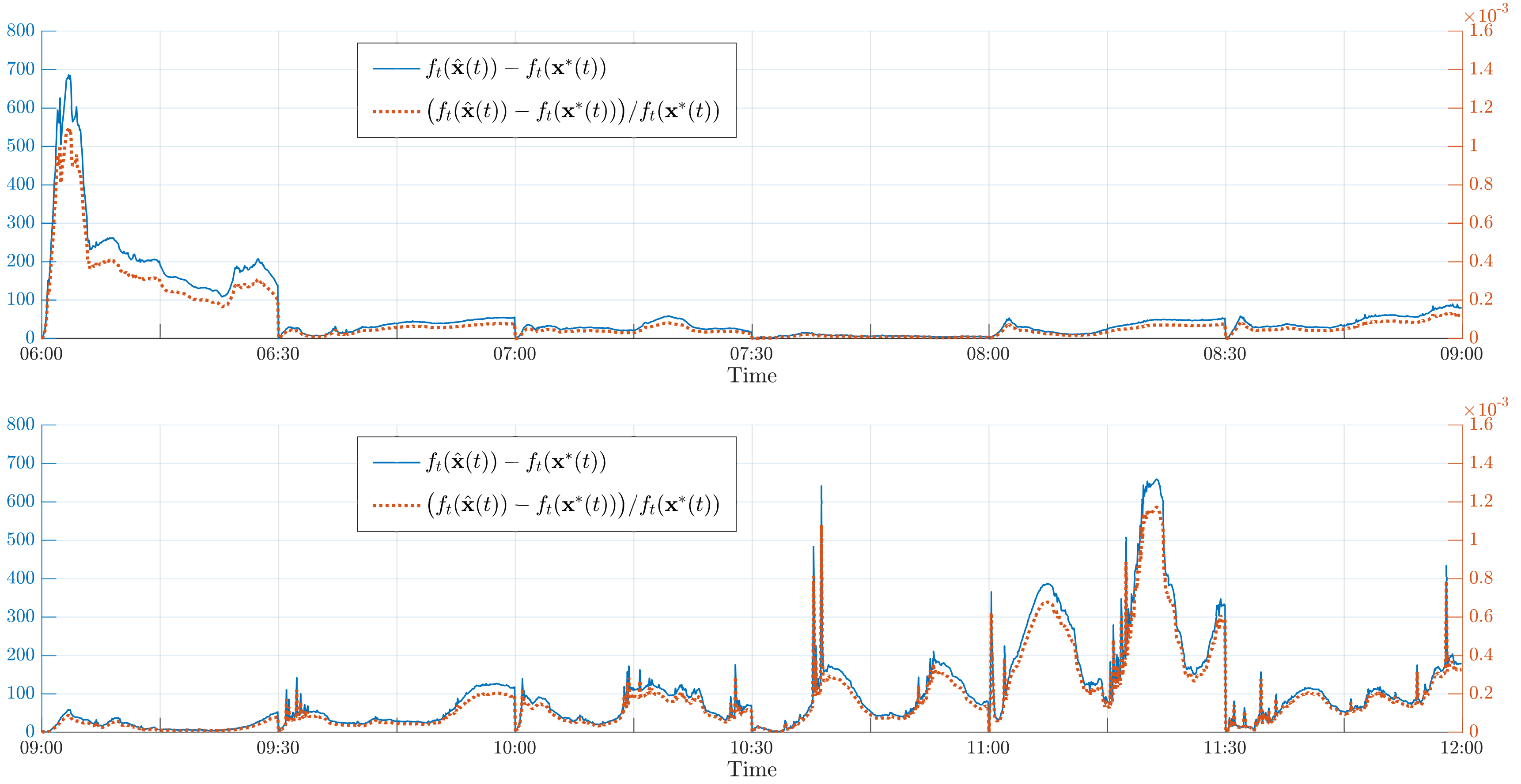
- IEEE 300 bus

Tracking performance



IEEE 300 bus

Tracking performance



IEEE 300 bus

Key message

Large network of DERs

- Real-time optimization at scale
- Computational challenge: power flow solution

Online optimization [feedback control]

- Network computes power flow solutions in real time at scale for free
- Exploit it for our optimization/control
- Naturally adapts to evolving network conditions

Examples

- Slow timescale: OPF
- Fast timescale: frequency control

Optimal placement

dealing with limited sensing/control

Guo (Caltech)

Guo & Low CDC 2017

Summary

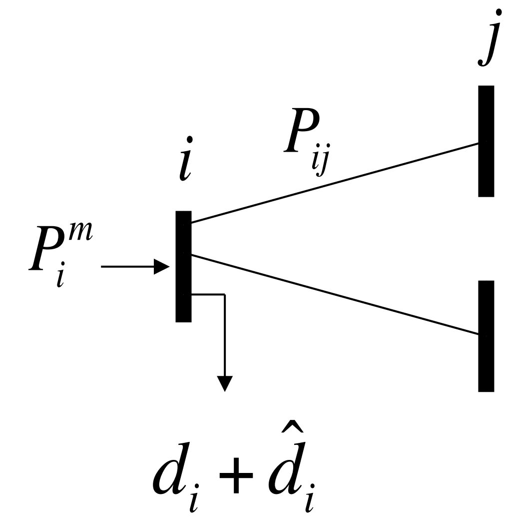
Characterization of controllability and observability

- of swing dynamics
- in terms spectrum of graph Laplacian matrix

Implications on optimal placement of controllable DERs and sensors

- set covering problem

Network model



swing dynamics:

$$-M_j\dot{\omega}_j = 1_{\mathcal{F}}(j)\hat{d}_j + 1_{\mathcal{U}}(j)d_j - P_j^m + \sum_{e\in\mathcal{E}} C_{je}P_e$$

$$\dot{P}_{ij} = B_{ij}(\omega_i - \omega_j)$$

controllable DER

$$y_j = 1_{\mathcal{S}}(j)\omega_j$$

frequency sensor

weighted Laplacian matrix

$$L = M^{-1/2}CBC^TM^{-1/2}$$

Algebraic coverage

spectral decomposition of L

$$L = Q\Lambda Q^T$$

eigenvectors of L

$$Q = \left[\beta_1 \ \cdots \ \beta_n \right]$$

algebraic coverage of bus j

$$\mathrm{cov}(j) := \left\{ s \;\middle|\; \beta_{sj} \neq 0 \right\}$$

Controllability

Theorem

Swing dynamics is controllable if and only if

- L has a simple spectrum   holds a.s.
- controllable DERs have full coverage

$$\bigcup_{j \in U} \mathrm{cov}(j) = \{\text{all buses}\}$$

Observability

Theorem

Swing dynamics is observable if and only if

- L has a simple spectrum   holds a.s.
- frequency sensors have full coverage

$$\bigcup_{j \in S} \mathrm{cov}(j) = \{\text{all buses}\}$$

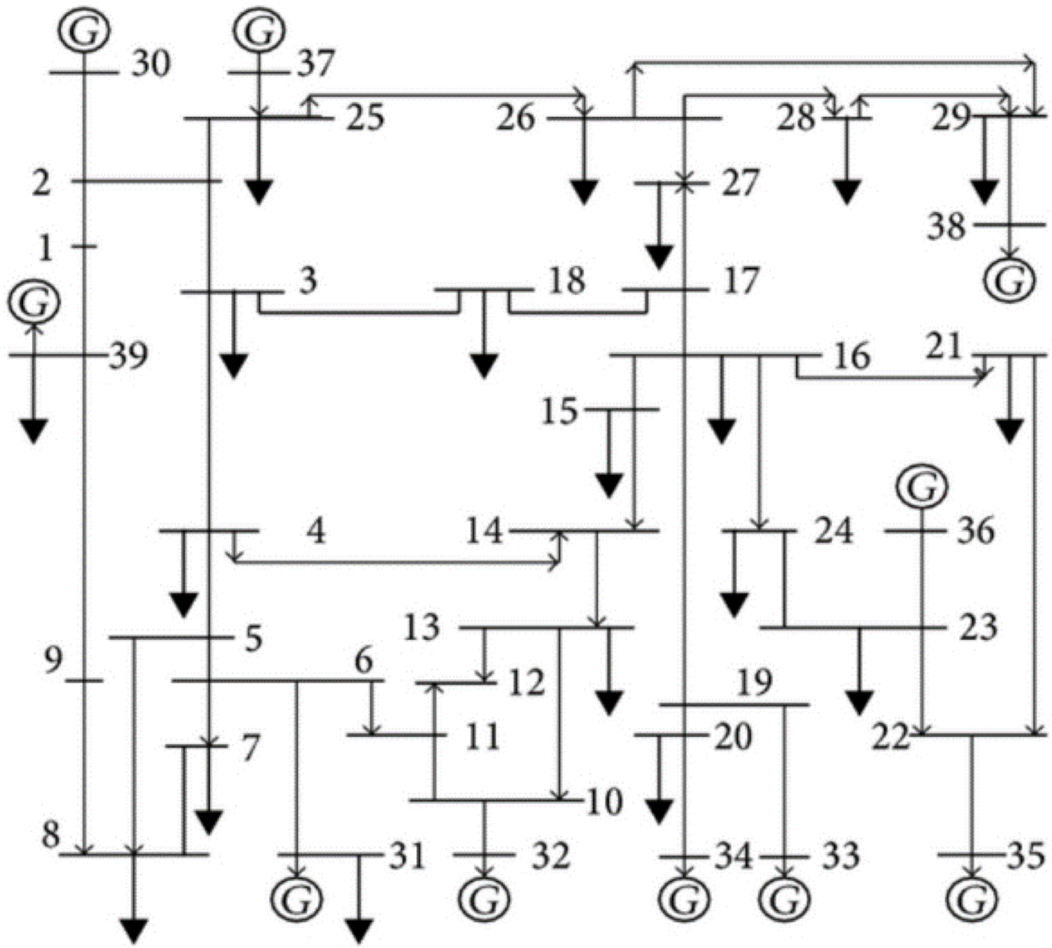
Application

Optimal placement of DER & frequency sensors

- set covering problem
- always install sensors at buses with controllable DERs, and vice versa

Example 2 – Control Coverage

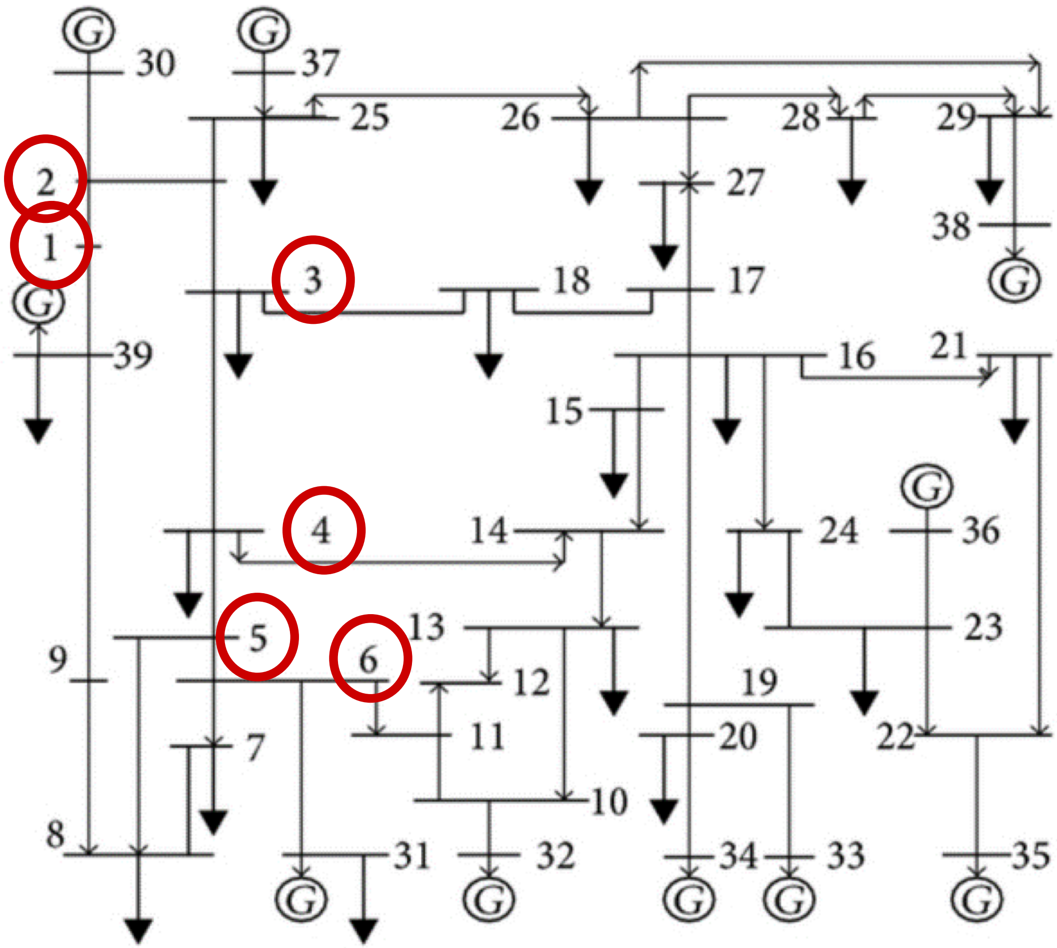
- Which choice provides controllability?



IEEE 39-bus
New England system

Example 2 – Control Coverage

- Which choice provides controllability?



(a) {1,2,3,4,5,6}

Example 2 – Control Coverage

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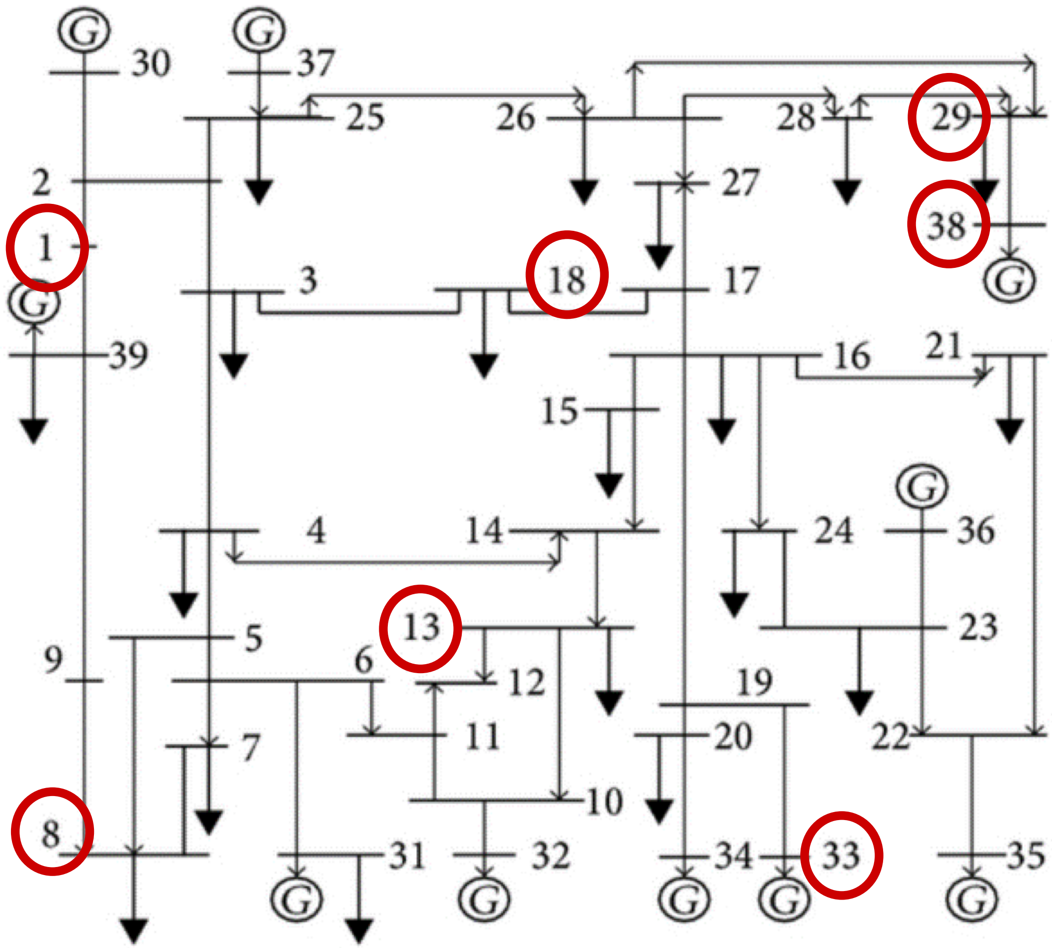
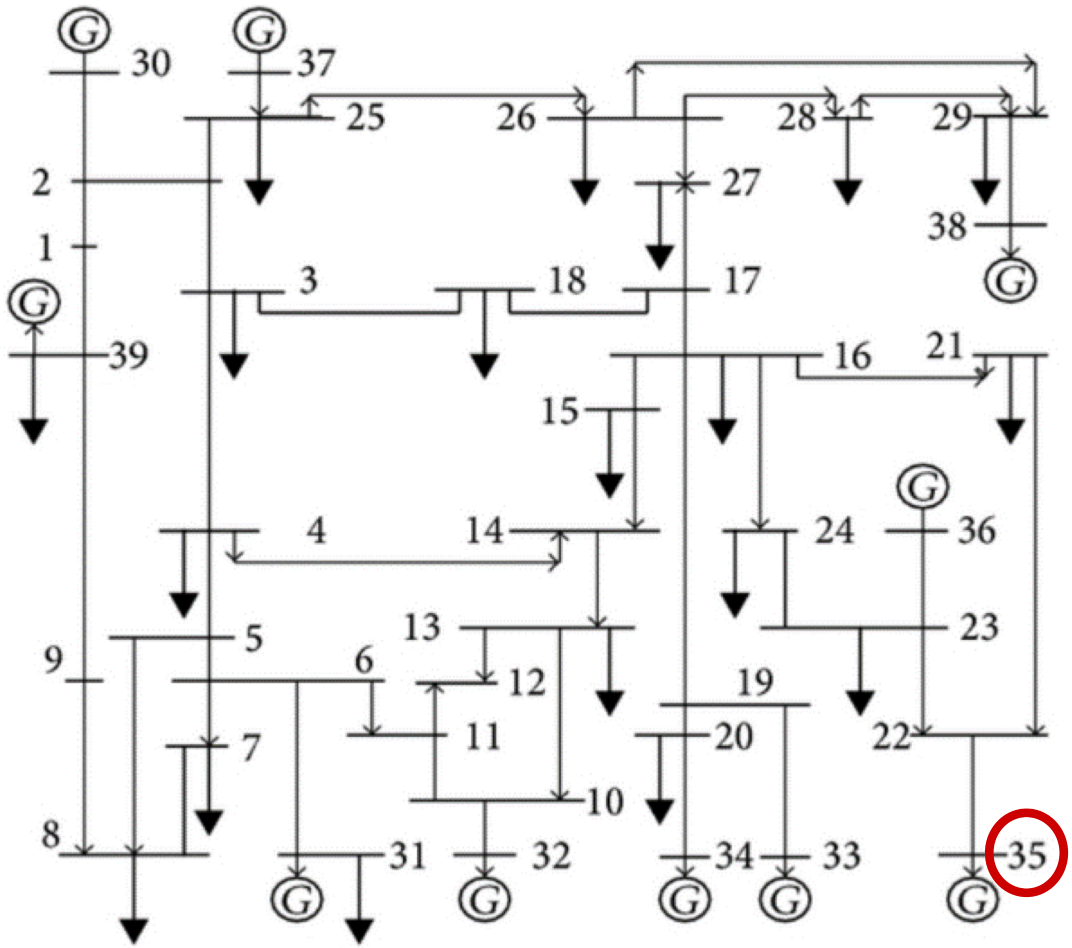


(a) {1,2,3,4,5,6}

(b) {1,18,13,8,29,33, 38}

Example 2 – Control Coverage

- Which choice provides controllability?



(a) {1,2,3,4,5,6}

(b) {1,18,13,8,29,33, 38}

(c) {35}

Example 2 – Control Coverage

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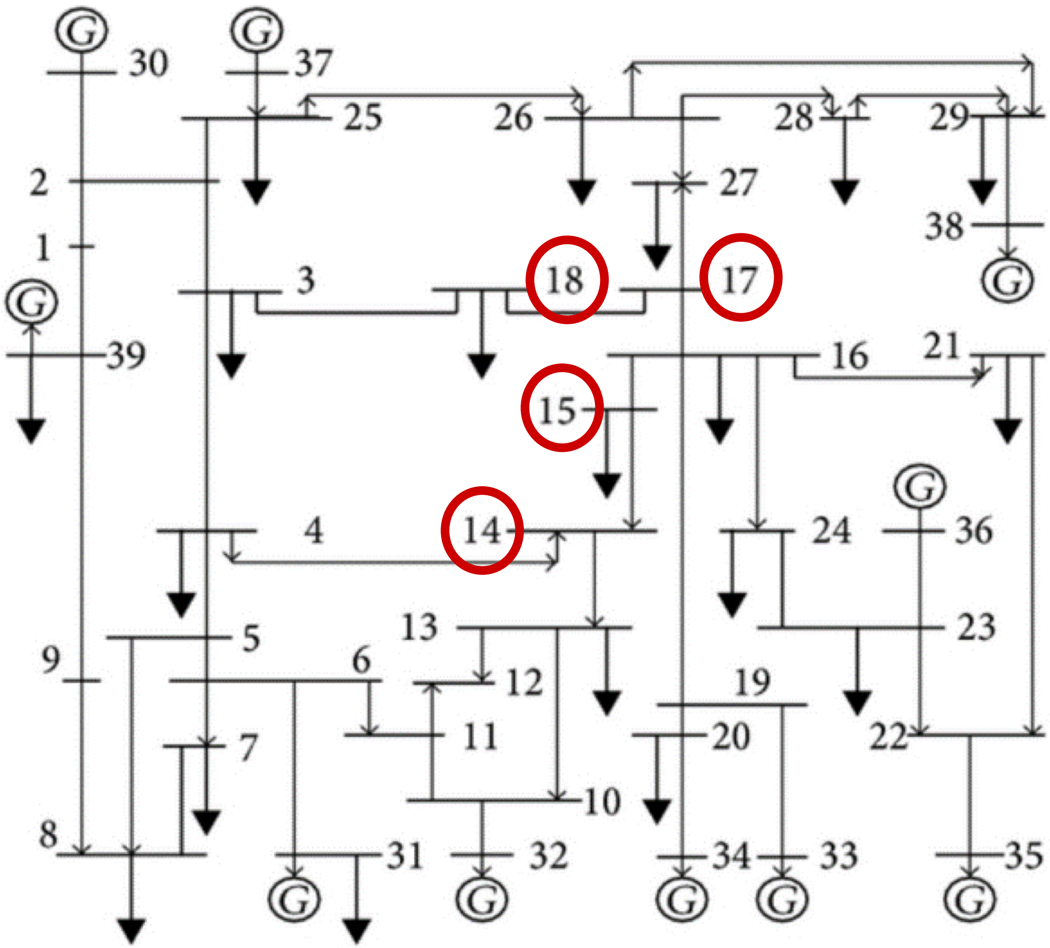

(a) {1,2,3,4,5,6}

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(c) {35}

(d) {14,15,17,18}

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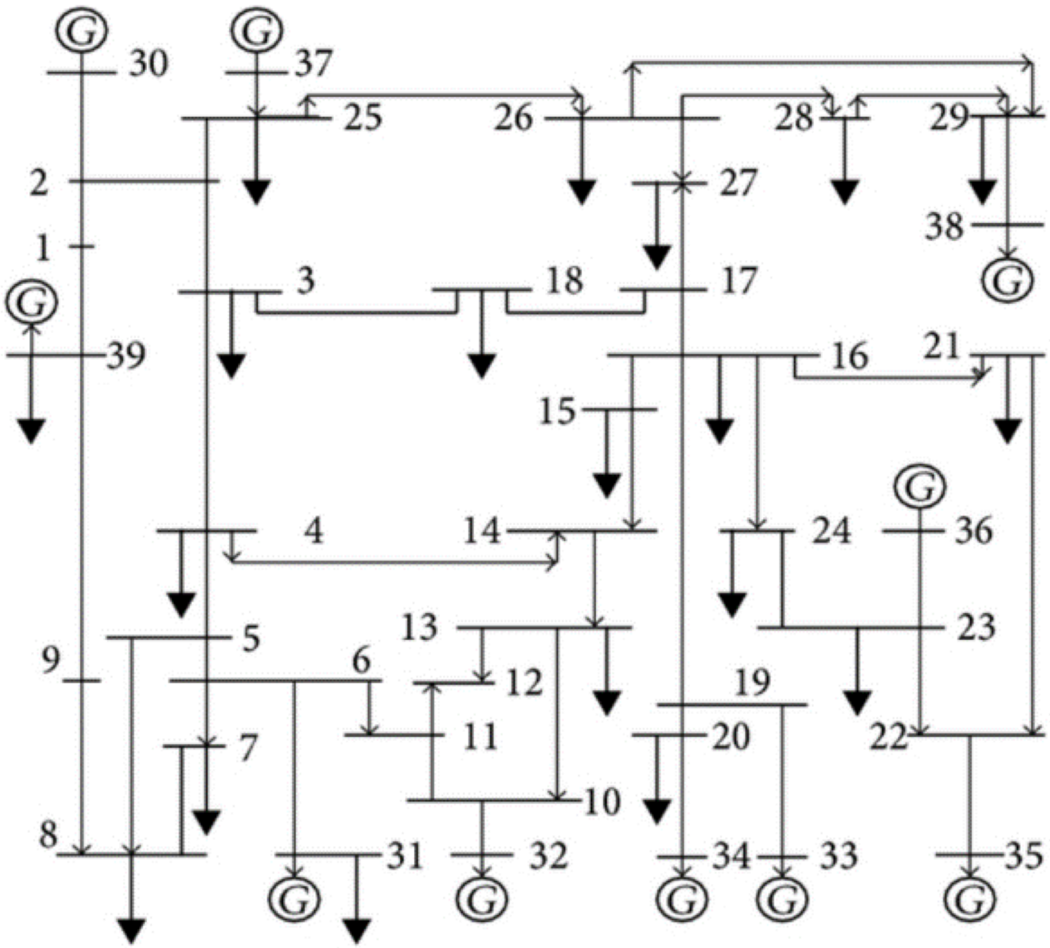


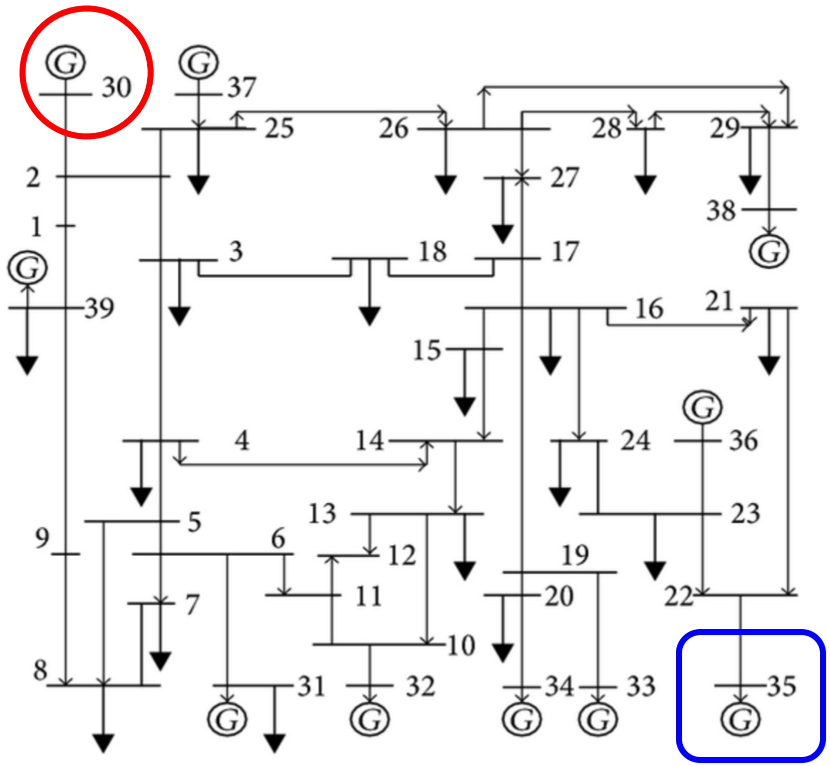
(a) {1,2,3,4,5,6}

(b) {1,18,13,8,29,33, 38}

(c) {35}

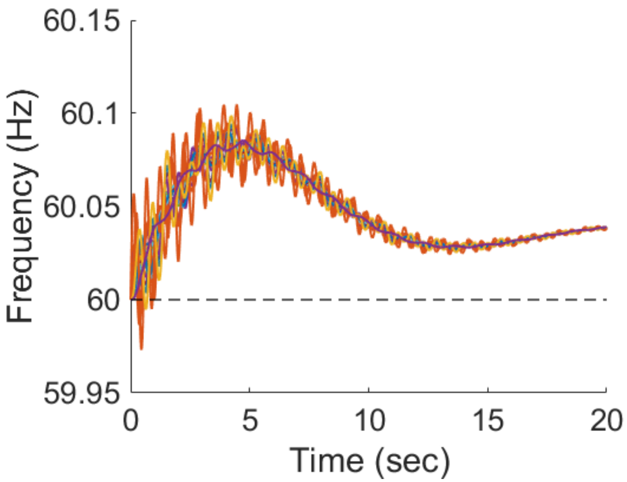
(d) {14,15,17,18}

Application

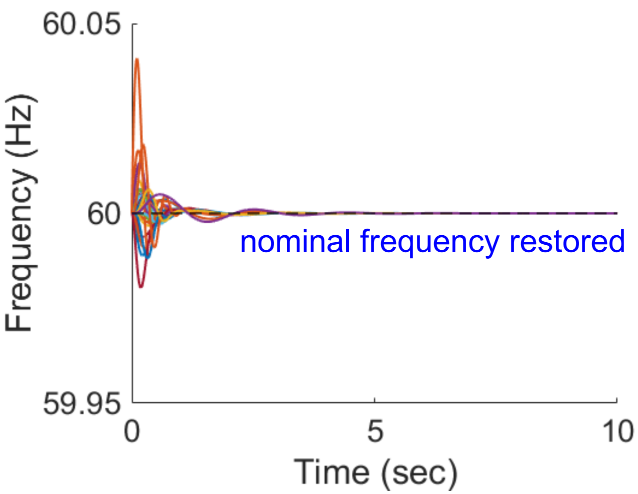


IEEE 39-bus New England system

1pu step disturbance at bus 30



without control



with local control at single bus 35

Summary

Relaxations of AC OPF

- Dealing with nonconvexity

Realtime AC OPF

- Dealing with volatility

Optimal placement

- Dealing with limited sensing/control