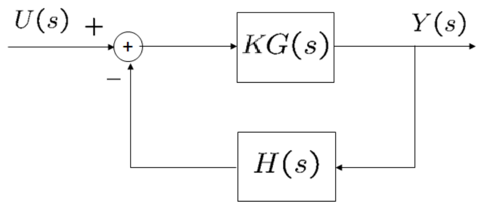

Defs. OLTF = $G(s)H(s)$; n_p, n_z = number of poles, zeros of OLTF; Characteristic Polynomial (CP) = $1 + KG(s)H(s)$

$$\Rightarrow \boxed{1 + KG(s)H(s) = 0 \Rightarrow K(s) = \frac{-1}{G(s)H(s)}}$$

Rule 1 RL is always symmetric with respect to **the real-axis—**remember that

Rule 2 RL has n branches, $n = n_p$

Rule 3 Mark poles (n_p) and zeros (n_z) of $G(s)H(s)$ with 'x' and 'o'

Rule 4 Each branch starts at OLTF poles ($K = 0$), ends at OLTF zeros or at infinity ($K = \infty$)

Rule 5 RL has branches on x-axis. These branches exist on real axis portions where the **total # of poles + zeros** to the right is an odd #

Rule 6 Asymptotes angles: RL branches ending at OL zeros at ∞ approach the asymptotic lines with angles:

$$\phi_q = \frac{(1+2q)180}{n_p - n_z} \text{ deg}, \forall q = 0, 1, 2, \ldots, n_p - n_z - 1$$

Rule 7 Real-axis intercept of asymptotes:

$$\sigma_A = \frac{\sum_{i=1}^{n_p} Re(p_i) - \sum_{j=1}^{n_z} Re(z_j)}{n_p - n_z}$$

Rule 8-1 RL branches intersect the real-axis at points where K is at an extremum for real values of s. Remember that:

$$\boxed{1 + KG(s)H(s) = 0 \Rightarrow K(s) = \frac{-1}{G(s)H(s)}}$$

We find the breakaway points by finding solutions (i.e., s^* solutions) to:

$$\frac{dK(s)}{ds} = 0 = -\frac{d}{ds}\left[\frac{1}{G(s)H(s)}\right] = 0 \Rightarrow \frac{d}{ds}\left[G(s)H(s)\right] = 0 \Rightarrow \text{ obtain } s^*$$

Rule 8-2 After finding s^* solutions (you can have a few), check whether the corresponding $K(s^*) = \frac{-1}{G(s^*)H(s^*)} = K^*$ is **real positive #**

Rule 8-3 **Breakaway pt.:** K^*_{max} (-ve $K''(s^*)$), **Break-in pt:** K^*_{min} (+ve $K''(s^*)$)

Rule 9 **Angle of Departure (AoD):** defined as the angle from a complex pole or Angle of Arrival (AoA) at a complex zero:

$$\text{AoD from a complex pole}: \ \phi_p = 180 - \sum_i \angle p_i + \sum_j \angle z_j$$

$$\text{AoA at a complex zero}: \ \phi_z = 180 + \sum_i \angle p_i - \sum_j \angle z_j$$

- $\sum_i \angle p_i$ is the sum of all angles of vectors to a complex pole in question from other poles, $\sum_j \angle z_j$ is the sum of all angles of vectors to a complex pole in question from other zeros
- '$\angle$' denotes the angle of a complex number

Rule 10 Determine whether the RL crosses the imaginary y-axis by setting:

$$1 + KG(s = j\omega)H(s = j\omega) = 0 + 0i$$

and finding the ω and K that solves the above equation. The value of ω you get is the frequency at which the RL crosses the imaginary y-axis and the K you get is the associated gain for the controller. You should obtain two equations (real = 0 and imaginary = 0) with two unknowns (K, ω). From there, you solve for K, ω pairs