# Enhancing Mobility with Quasi-passive Variable Stiffness Exoskeletons

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***Abstract*—Shoes were invented to provide user comfort using rubber soles, despite marginal improvement in human mobility. Unlike shoes, current lower-limb exoskeletons use fixed stiffness springs to store and recycle energy to improve mobility. However, the maximum kinetic energy a human can accumulate when augmented with a fixed stiffness spring is limited by the maximum deflection and the ability of the limbs to generate force; tying the advantage of fixed stiffness exoskeletons to biological constraints. Here we propose a *method of improving mobility using quasi-passive variable stiffness spring exoskeletons* where the maximum kinetic energy accumulated by a human is independent of the limb deflection and the ability of the limb to generate force. This is achieved by a *variable stiffness augmentation of the human where the exoskeleton does not provide mechanical work*. The theoretical advantage provided by this new augmentation method can be useful in demanding tasks where humans could benefit from increased speed and reduced energy cost of motion.**

***Index Terms*—Exoskeletons, Variable stiffness augmentation, Quasi-passive stiffness modulation.**

## I. Introduction

Given limited size, force capability, and metabolic rate of human limbs, how to maximize mobility [1]–[4]? Most mobile animals such as kangaroos, frogs, and fleas have long and muscular limbs to do more work when exerting force against the ground to move faster [5]–[10]. However, mimicking Nature's evolutionary solution in humans requires altering limb geometry and augmenting muscular force, using active devices with large motors and heavy power supplies [11]–[14].

An alternative method to augment the force of the limb is by placing a spring in series or in parallel with the limb [15]–[17]. The spring augments mobility by storing and releasing energy in cyclic zero output work tasks [18]–[23], or by rapidly releasing energy faster than the un-augmented limb could do [24], [25]. A low stiffness spring in parallel with the legs can increase the ground contact time and the output work of the limb. A low stiffness spring could also be preloaded by the human in the air, and then used to provide force on the ground [26]. However, a low stiffness spring cannot store considerable energy within the limited deflection of the limb. A high stiffness spring could store more energy for the same deflection, but achieving the same deflection requires either large force or small but precisely timed resonant forcing. In both cases, the limb is severely limited by the inverse relation between the velocity and the maximum limb force [27], [28].

In this paper, we propose a novel method to increase human mobility using a variable stiffness exoskeleton in parallel with the legs to accumulate the kinetic energy of a human without doing mechanical work. This is done by enabling the human to do work in the air, and by changing the stiffness of the exoskeleton [29] to support the body on the ground. The key advantage of the variable stiffness augmentation over fixed stiffness augmentation is that it maximizes the work done by the human within the biological limitation of the legs.

The working principle of a variable stiffness exoskeleton may be understood by considering a jumping task [1], [30], see Fig. 1. In order to jump, the augmented human (gray) first preloads a fixed stiffness spring (green) with the legs (red) by doing work in the air, see Fig. 1a(A). The spring is preloaded during the extension of the legs as permitted by the design of the exoskeleton, see Fig. 1a(A). The work done by the legs is then locked inside the spring until the human is at the lowest position upon landing, see Fig. 1a(B-C). Subsequently, the preloaded spring (green) is unlocked to exert a force to the ground unaffected by the biological limitations of the legs, see Fig. 1a(C). On the ground, the variable stiffness spring (blue) first stores the kinetic energy of the preceding motion Fig. 1a(B), and then releases its energy by applying force to the ground, see Fig. 1a(C). Repeating this action may allow the human to accumulate kinetic energy through multiple jumps.

The stiffness of the exoskeleton is adjusted in the air between jumps because doing so requires no mechanical work; the variable stiffness spring (blue) stores no energy in the air, see Fig. 1a(A). The variable stiffness spring is set to the minimum stiffness to store the kinetic energy of the body within the deformation limit of the legs, see Fig. 1(B). The added benefit of the minimum stiffness is that it maximizes the ground contact time Fig. 1(B-C), and thereby minimizes the effect of an inevitable time delay when unlocking the preloaded spring (green), see Fig. 1(C). As a result, the human could use the variable stiffness exoskeleton to reach a desired jump height, and kinetic energy, in fewer jumps compared to the best fixed stiffness exoskeleton.

An alternative to the parallel variable stiffness augmentation is the series variable stiffness augmentation method [31]. Series elasticity has the theoretical advantage of avoiding kinetic energy losses by preventing the feet from colliding with the ground. However, in the series arrangement, the maximum average force required by the human to do work scales with the energy stored in the spring and the kinetic energy of the motion. Consequently, when the human is augmented with

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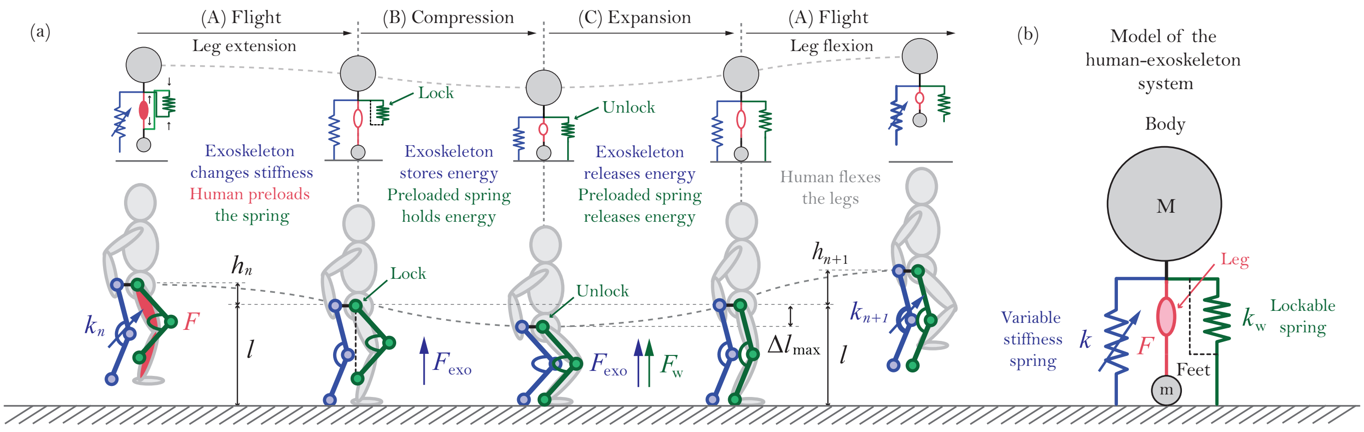


Fig. 1. Variable stiffness augmentation. (a) Jumping. (b) Minimalistic model of the human-exoskeleton system.

series elasticity, the maximum attainable kinetic energy is limited by the maximum average force capability of the legs. This is not the case in the proposed parallel variable stiffness augmentation method becasue here any force used to compress the spring in the air would do positive work on the ground.

In summary, the proposed quasi-passive variable stiffness augmentation method bypasses fundamental geometric and force limitations of fixed stiffness exoskeletons, enabling the human to accumulate kinetic energy over repeated leg movements. The paper proves the advantage of quasi-passive variable stiffness augmentation compared to fixed stiffness augmentation, discusses how to realize this augmentation, and presents the theoretical limits of this new augmentation method. While the development of *quasi-passive variable stiffness devices* remains an engineering challenge, these devices may pave the way towards mobility enhancing exoskeletons.

## II. Quasi-passive Variable Stiffness Augmentation

Human augmentation methods may be explored by building exoskeleton devices for experimental investigation, see Section IV. However, theoretical investigations of previously unexplored augmentation methods may also be useful in reducing trial and error exploration via experimentation. Similar to previous works [2]–[4], [30], [32]–[41], here we use a minimalistic model and a theoretical investigation to propose a novel means of human augmentation.

In order to investigate the utility of quasi-passive variable stiffness augmentation in enhancing human mobility, we consider hopping, which is one of the simplest motions involving repeated work input by the legs. This motion may be seen as a building block of a more complex running motion, but may also be useful to accumulate energy in the exoskeleton to leap across a great distance.

Hopping may be modeled using a spring-mass system [30], [42]. The essential dynamics of the human-exoskeleton system can be represented by the feet (mass $m$) and the body (mass $M$), connected by the exoskeleton (a variable stiffness spring $k$ (blue) and a fixed stiffness spring $k_W$ (green)) in parallel with the leg (force generating actuator $F$ (red)), see Fig. 1b. In this model, the springs possess the same deformation on the ground, while the variable stiffness spring is not deformed in the air. We use this model to investigate the benefits of the proposed quasi-passive variable stiffness augmentation method.

For the purpose of the subsequent analysis, we characterize human mobility by maximum speed or kinetic energy, acceleration (the time required to achieve a desired speed or kinetic energy), and the energy efficiency of motion. In this Section, we present the *theoretical limits* of the augmented human motion. These limits indicate that an augmented human could theoretically (i) attain more than fifty times larger kinetic energy compared to an un-augmented human (Subsection II-A), (ii) reach the maximum kinetic energy in less than half the time and half the number of jumps (Subsection II-B), and (iii) build up energy more than fifty percent more efficiently compared to the best fixed stiffness exoskeleton (Subsection II-C). Although, there are significant practical challenges in approaching these theoretical limits (Section III), our predictions suggest that a variable stiffness exoskeleton could provide significant mechanical advantage for a human user despite *providing no mechanical work.*

### A. Maximum Jump Height and Maximum Kinetic Energy

Although humans can jump as many times as they have energy for, the maximum attainable kinetic energy is the work of a single jump, since landing dissipates almost all the energy of the previous jump [43]. This is not the case when a spring is used in parallel with the legs to store energy. However, the maximum amount of energy that can be stored by a fixed stiffness spring is bounded by human geometry:

$$\Delta l \in [0, \Delta l_{\max}] \tag{1}$$

where the maximum contraction of the leg $\Delta l_{\max}$ may be half of the total leg length in jumping [44], see Table I. As subsequently shown, this geometric constraint precludes the human to accumulate considerable energy even if augmented with *the best fixed stiffness exoskeleton.*

In order to recover all the energy during landing within the geometric constraint of the leg (1), the stiffness of the exoskeleton should be adjusted according to:

$$k(h) = k_0\left(1 + \frac{h}{\Delta l_{\max}}\right) \tag{2}$$

where $h$ is the initial drop height, while $k_0 = 2Mg/\Delta l_{\max}$ is the baseline stiffness to support the human when the body is at the lowest position, see Fig. 1a(B-C). The stiffness of the exoskeleton can be set according to (2) in flight when the variable stiffness spring is at equilibrium length, see Fig. 1a(A). In doing so, *the exoskeleton does not do any work*; it behaves as a passive spring that can be used to recycle energy during ground contact. Therefore, all the energy for the motion is provided by the human and not the variable stiffness exoskeleton.

After the exoskeleton fully decelerates the motion, the human will start to rebound and the preloaded fixed stiffness spring will be unlocked to exert a force and do work i.e., increase the kinetic energy of the body, see Fig. 1a(C). The work done by the spring on the ground is the work done by the human on the spring in the preceding flight phase. We assume that the work done by the human is bounded by the maximum work done in a single un-augmented jump:

$$W \in [0, W_{\max}] = [0, \bar{F}\Delta l_{\max}] \tag{3}$$

where we calculate $W_{\max} = (M + m)gh_{\max} + Mg\Delta l_{\max}$ using the measured weight and the observed maximum jump height $h_{\max}$ of gymnasts [44] while $\bar{F} = W_{\max}/\Delta l_{\max}$ is the average force of the leg, see Table I.

Assuming that the stiffness $k_n = k(h_n)$ follows the control strategy given by (2), and that the preloaded spring is released at the lowest point upon landing, see Fig. 1a(B-C), the peak vertical height that could be reached during the subsequent flight phase is:

$$h_{n+1} = \frac{M}{M+m}h_n + \frac{W}{(M+m)g} \tag{4}$$

where $h_n$ is the previous jump height while $0 < M/(M + m) < 1$ indicates the incomplete energy recovery due to collisional energy losses. Similarly, the kinetic energy of the human-exoskeleton system after each jump is given by:

$$E_{n+1} = \frac{M}{M+m}E_n + W \tag{5}$$

where $E_n$ is the kinetic energy just before landing. According to (4) and (5), the maximum jump height $h_\infty$ and the saturation kinetic energy $E_\infty$ are given by:

$$\frac{h_\infty}{\Delta l_{\max}} = \frac{W}{mg\Delta l_{\max}} \quad \text{and} \quad \frac{E_\infty}{W} = 1 + \frac{M}{m}. \tag{6}$$

The theoretical advantage provided by the variable stiffness exoskeleton can be predicted by comparing the maximum jump height and the maximum kinetic energy reachable (i) with the exoskeleton

$$\left.\frac{h_\infty}{\Delta l_{\max}}\right|_{W=W_{\max}} = 41 \quad \text{and} \quad \left.\frac{E_\infty}{W}\right|_{\forall W\in(0,W_{\max}]} = 25 \tag{7}$$

and (ii) without the exoskeleton

$$\frac{h_\infty}{\Delta l_{\max}} = \frac{17}{25} \quad \text{and} \quad \frac{E_{\text{no exo}}}{W_{\max}} = \frac{17}{41}, \tag{8}$$

where the calculations assume physical parameters measured from the jumping of gymnasts given in Table I.

TABLE I
PHYSICAL PARAMETERS OF GYMNASTS REPORTED IN [44].

| Description | Symbol | Value | Units |
|---|---|---|---|
| Body mass | $M+m$ | 72 | kg |
| Leg length | $l$ | 0.99 | m |
| Gravitational acceleration | $g$ | 9.81 | $\mathrm{ms}^{-2}$ |
| Feet mass | $m/(M+m)$ | 0.04 | – |
| Maximum jump hight | $h_{\max}/l$ | 0.34 | – |
| Maximum leg contraction | $\Delta l_{\max}/l$ | 0.5 | – |
| Average force of the leg | $\bar{F}/(Mg)$ | 1.7 | – |
| Maximum work | $W_{\max}/(Mgl)$ | 0.85 | – |

Expressions (7) and (8) suggest more than fifty times higher peak jump height and accumulated kinetic energy of an augmented human compared to an un-augmented human. However, in order to reach saturation, the variable stiffness exoskeleton should have a large enough stiffness range:

$$k(h_n) \in [k_0, k_\infty] \quad \text{where} \quad \frac{k_\infty}{k_0} = 1 + \frac{W_{\max}}{mg\Delta l_{\max}} = 42 \tag{9}$$

and should be able to store enough energy to reach saturation:

$$\frac{1}{2}k_\infty \Delta l_{\max}^2 \geq \frac{M}{M+m}E_\infty. \tag{10}$$

Figure 2 shows the jump height, kinetic energy, stiffness, and the average force of the leg for the jumping motion with the variable stiffness exoskeleton (where the human uses the preloaded spring to exert force on the ground), compared to four conventional fixed stiffness parallel leg exoskeletons (where the human does work by using the legs to exert force against the ground) [22]. In order to present the theoretical limits for augmented jumping, we assume that the work done by the legs in every jump is the work of the legs in a single un-augmented jump given in Table I:

$$W = W_{\max}.$$

We note that $W_{\max}$ is calculated from experimental data and as such it factors in biological limitations that include the position and velocity dependent force of the leg in natural jumping.

Figure 2(b) shows that the lowest fixed stiffness exoskeleton $k = k_0$ (blue line) provides the same increase of energy as the variable stiffness exoskeleton (black line) in the first jump. However, the lowest fixed stiffness exoskeleton cannot store and accumulate energy beyond a single jump because it loses any additional energy in the collision of the two halves of the legs during ground contact, see Fig. 2(a). Conversely, the variable stiffness exoskeleton increases stiffness after each jump to recover more energy in every next jump, see Fig. 2(a-c). According to Fig. 2(a), the best fixed stiffness exoskeleton optimized for the current world record jump height $h = 2.45$ m (gray line), reaches that height in five jumps while the variable stiffness exoskeleton reaches the same height in three jumps (black line). The variable stiffness exoskeleton reaches the same height in fewer jumps because it maximizes the energy provided by the human in each jump, see Fig. 2(b). The variable stiffness exoskeleton maximizes the energy provided by the human by changes stiffness Fig. 2(c) to increase the distance for the preloaded spring to apply force. It may be also noted that unlike the variable stiffness exoskeleton, the fixed stiffness exoskeleton optimized for this task cannot be used to jump higher than $h = 2.45$ m.

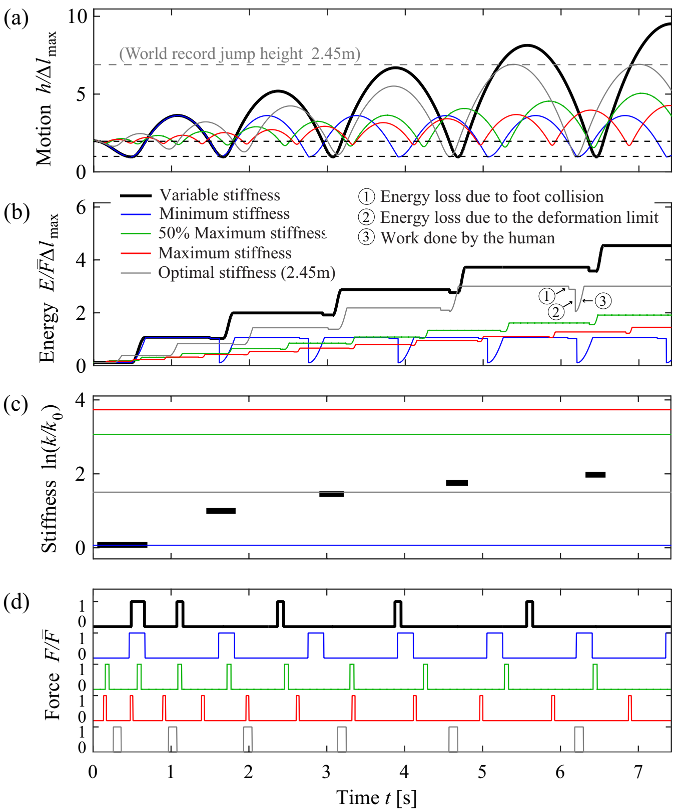


Fig. 2. Comparison between the proposed variable stiffness augmentation method and four different fixed stiffness augmentation methods. The figure shows: (a) the jump height, (b) energy, (c) stiffness, and (d) the leg force for the variable stiffness (black), the lowest fixed stiffness $k = k_0$ (blue line), medium fixed stiffness $k = \frac{1}{2}k_\infty$ (green line), high fixed stiffness $k = k_\infty$ (red line) and the fixed stiffness exoskeleton optimized for the current world record jump height $h_\infty = 2.45$ m (gray).

We will subsequently show that the highest fixed stiffness exoskeleton $k = k_\infty$ Fig. 2 (red line) can reach the same saturation energy $E_\infty$ and jump height $h_\infty$ as the variable stiffness exoskeleton (black line) because both have the same energy storing capacity. However, the variable stiffness exoskeleton can be used to accumulate energy considerably faster and using fewer jumps (Section II-B) while requiring less work to be done by the human (Section II-C).

### B. Energy Accumulation Rate

Acceleration is a defining feature of mobility, where fewer jumps and less time taken is desirable to reach a target height and energy. Next we show that, a variable stiffness spring exoskeleton accumulates energy faster than a fixed stiffness spring exoskeleton; it reaches the saturation energy $E_\infty$ in nearly half the time and half the number of jumps compared to the best fixed stiffness exoskeleton.

Figure 3 shows the jump height and the energy accumulation predicted by the model when the jumping motion starts from a stationary ground position. According to the figure, the variable stiffness exoskeleton could enable the human to reach 99% of the saturation height in 390 s (black line) compared to the 880 s (red line) when the human uses the high fixed stiffness exoskeleton $k = k_\infty$. In this case, the high fixed stiffness exoskeleton is the only fixed stiffness exoskeleton that has the energy storing capacity to reach the saturation height $h_\infty$ of the variable stiffness exoskeleton.

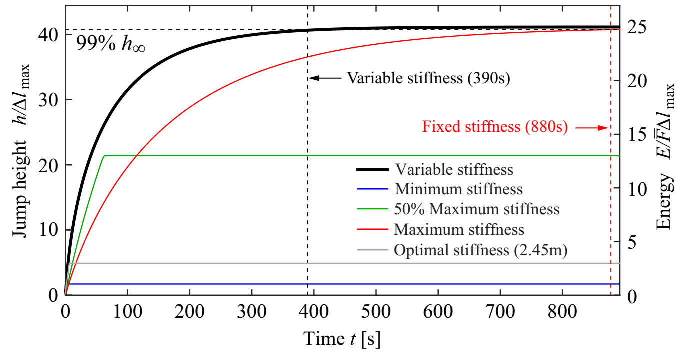


Fig. 3. Jump height and energy versus time of jumping with the variable stiffness exoskeleton (black) versus different fixed stiffness exoskeletons (blue, green, red, and gray). The calculations were done using the physical parameters of gymnasts given in Table I.

Figure 4 shows the jump height and the accumulated energy for different number of jumps. We observe that the variable stiffness exoskeleton reaches 99% of the saturation height $h = 0.99h_\infty$ in less than half the number of jumps $N$, compared to the best fixed stiffness exoskeleton $N_F$, as predicted by (5):

$$N = -\frac{\ln\left(1 - E/E_\infty\right)}{\ln\left(1 + m/M\right)} \approx 112 < N_F \approx 255. \tag{11}$$

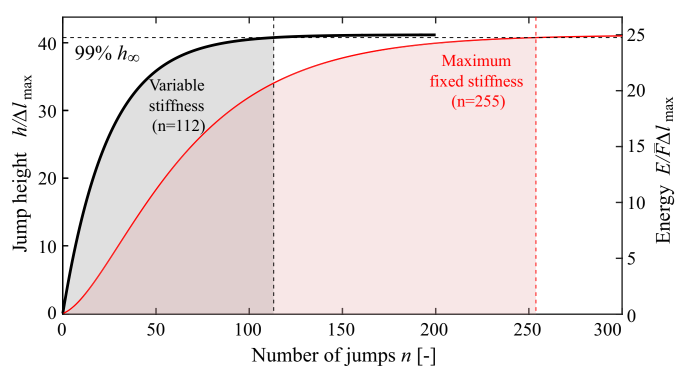


Fig. 4. Jump height and energy versus the number of jumps for the variable stiffness exoskeleton (black) and the highest fixed stiffness exoskeleton (red).

The variable stiffness exoskeleton accumulates energy faster, and in fewer jumps, because the work done by the legs in each jump is the maximum work the legs can do in a single jump

$$W = W_{\max} = \bar{F}\Delta l_{\max}, \tag{12}$$

given the average force $\bar{F}$ and the maximum deflection $\Delta l_{\max}$ taken from un-augmented jumping, see Table I.

When using a fixed stiffness exoskeleton, the deflection of the leg $\Delta l_F$ depends on the stiffness of the spring $k_F$ and the energy stored by the spring $E$, and is almost always smaller than the maximum deflection of the leg:

$$\Delta l_F \in [0, \Delta l_{\max}]. \tag{13}$$

In particular, for the fixed stiffness augmentation, the deflection of the leg is the smallest at the beginning of the motion and only reaches its maximum when the kinetic energy can no longer increase:

$$\left.\frac{\Delta l_F}{\Delta l_{\max}}\right|_{E\to 0} \approx \frac{k_0}{k_F} \quad \text{and} \quad \left.\frac{\Delta l_F}{\Delta l_{\max}}\right|_{E\to E_\infty} \approx 1. \tag{14}$$

Consequently, the work done by the legs using a fixed stiffness exoskeleton $W_F = \bar{F}\Delta l_F$ cannot exceed the work done by the legs when using the variable stiffness exoskeleton:

$$\frac{W_F}{W_{\max}} = \frac{\Delta l_F}{\Delta l_{\max}} \in \left[\frac{k_0}{k_F}, 1\right]. \tag{15}$$

The legs do less work in the fixed stiffness case compared to the variable stiffness case in almost all jumps, except when the fixed stiffness exoskeleton is set to the lowest stiffness $k_F = k_0$. However, if $k_F = k_0$ then the fixed stiffness exoskeleton cannot build up kinetic energy because it cannot store energy beyond that of a single jump (see Fig. 2 and Fig. 3 blue lines).

In summary, the variable stiffness exoskeleton allows the human to accumulate energy in the least time and the least number of jumps for any energy and final jump height, unlike a fixed stiffness exoskeleton which is only optimal for a specific energy and jump height defined by the fixed stiffness spring.

### C. Energy Efficiency

It is desirable to maximize energy efficiency in order to minimize the total work done by the human during the motion. Next we show that the variable stiffness exoskeleton provides more efficient energy accumulation compared to fixed stiffness exoskeletons for any saturation energy and jump height. This means that the human could do less work to reach the same jump height.

The efficiency of a single jump is defined by the change of the kinetic energy $\Delta E_n = E_{n+1} - E_n$ and the work done by the human $\bar{F}\Delta l$:

$$\eta_n = \frac{\Delta E_n}{\bar{F}\Delta l} = 1 - \frac{m}{M+m}\frac{E_n}{\bar{F}\Delta l}. \tag{16}$$

According to (16), the efficiency of each individual jump is maximized by maximizing the deflection of the limb:

$$k_n = k(h_n) \Rightarrow \Delta l = \Delta l_{\max} \;\Rightarrow\; \eta_n = \eta_{n\max}. \tag{17}$$

This is only possible using the variable stiffness exoskeleton, where the stiffness is adjusted according to (2).

The total efficiency $\eta$ is the ratio between the total accumulated energy $E$ and the work done by the human because the exoskeleton does no mechanical work. For the variable stiffness exoskeleton the total efficiency is:

$$\eta = \frac{E}{N\bar{F}\Delta l}. \tag{18}$$

The total efficiency to reach 99% of the saturation height is proportional to the area under the curves shown in Fig. 4, and can be numerically calculated. For example, using the human data in Table I, reaching 99% of the saturation height results in a total efficiency of $\eta \approx 22\%$ for the variable stiffness exoskeleton and $\eta_F \approx 12\%$ for the fixed stiffness exoskeleton. These two efficiencies indicate that the variable stiffness exoskeleton is more efficient $\eta/\eta_F \approx 1.8$. We note that in order to reach the current world-record high jump ($h_n = 2.45$m), the energy efficiency of the variable stiffness exoskeleton is still considerably higher than the efficiency of the best fixed stiffness exoskeleton designed for this particular task: $\eta/\eta_F \approx 1.1$.

## III. Practical Limitations

Our theoretical analysis relies on the human applying force at certain times and an exoskeleton capable of changing stiffness. In this analysis, we have assumed that (i) the average force of the legs $\bar{F}$ in a single jump remains constant through repeated jumps despite the increased speed of the motion, (ii) the preloaded spring is unlocked with exact timing, (iii) the exoskeleton is massless, (iv) the exoskeleton can change stiffness in a large range with negligible energy cost, and that (v) the human body and the exoskeleton can be represented by a simple model; the internal degrees of freedom in the human body are neglected and the exoskeleton is modeled with linear springs. Here we explore how physical limitations including the velocity-dependent leg force [27], [28], [45], [46], imperfect timing [47], [48], added mass of the exoskeleton, stiffness range limitation [49], energy cost of stiffness modulation [50], complexity of the human body [51], and the nonlinearity of the springs may affect the aforementioned claims.

Figure 5 and Fig. 6 show the combined effect of the (i) velocity dependent average force of the leg (detailed in Section III-A), (ii) time delay in releasing the preloaded spring (detailed in Section III-B), and the (iii) added mass of an ideal exoskeleton (detailed in Section III-C). In order *not* to favor the variable stiffness augmentation method in the comparison, we assume that the work done by the human in the air is half the work of a single un-augmented jump:

$$W = \frac{1}{2}W_{\max},$$

despite a longer time between subsequent jumps to do work in the air compared to the time available on the ground. We also assume that the human can apply force on the ground with exact timings when using the fixed stiffness exoskeletons.

Figure 5 predicts the energy and jump height that could be reached by the human using the variable stiffness exoskeleton and various fixed stiffness exoskeletons. Unlike in the ideal case shown in Fig. 3, the maximum height and energy reachable by the best fixed stiffness exoskeleton is $h = 2.15$ m (red line), which is significantly below the maximum height and energy reachable using the variable stiffness exoskeleton $h = 5.0$ m (black line), see Fig. 5. Under the considered practical effects, the variable stiffness exoskeleton can still reach the current world record jump height $h = 2.45$ m in

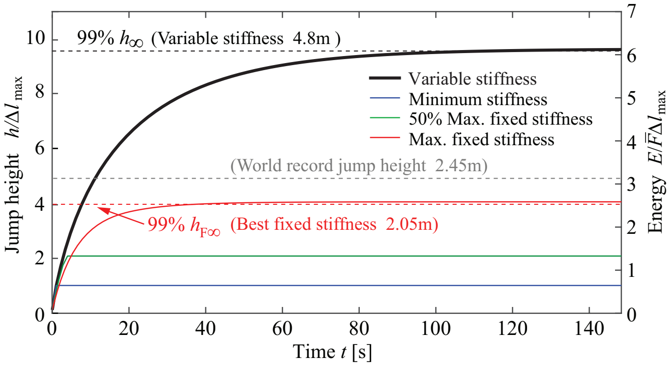


Fig. 5. Jump height and the energy given the velocity-dependent leg force, the time-delay in applying force on the ground, and the mass of an ideal exoskeleton, as defined in Sections III-A-III-C.

ten jumps, while the fixed stiffness exoskeleton is unable to reach the same height.

Figure 6 shows the jump height, energy, stiffness, and the average force by the legs for the first five jumps. We observe that: (i) the variable stiffness exoskeleton changes stiffness to keep the deflection of the legs fixed in every jump, see Fig. 6(c,a) (black line), (ii) the variable stiffness exoskeleton maximizes the work done by the preloaded spring, see the energy increments per jump in Fig. 6(b) (black line), (iii) the work and the average force of the leg are independent of the movement speed when the human does work in the air, see Fig. 6(b,d) (black line), and (iv) the work and the average force of the leg decrease with increased movement speed when the human does work on the ground, see Fig. 6(b,d) (red, green and blue lines).

In summary, comparison of Figs. 2, 3 and Figs. 5, 6 shows that under the considered practical limitations, the maximum jump height and the energy accumulated for a given number of jumps is reduced as expected, while the benefit of the proposed variable stiffness augmentation method has considerably increased compared to the fixed stiffness augmentation.

### A. Velocity-dependent Force

In Section II, we assumed that the human exerts a constant average force in jumping, even though muscular force has been observed to decrease with higher movement velocities [27], [28], [45]. For an individual muscle, force exerted is inversely proportional to muscle velocity [27], while the net force of the entire leg may be assumed linear with respect to the extension velocity in jumping [28], [45]:

$$\frac{F}{\bar{F}} \in \begin{cases} 1 - \frac{\Delta \dot{l}}{\Delta \dot{l}_{\max}} & \text{if } 0 \le \frac{\Delta \dot{l}}{\Delta \dot{l}_{\max}} \le 1 \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

where we assume that $\Delta \dot{l}_{\max} \approx 6.3$ m/s is the approximate velocity at which the force is zero [46].

When humans exert force against the ground to jump higher, the expansion velocity of the legs increases. Faster expansion decreases the average force, see (19) and Fig. 6(d) (red and green lines). Consequently, faster expansion decreases the

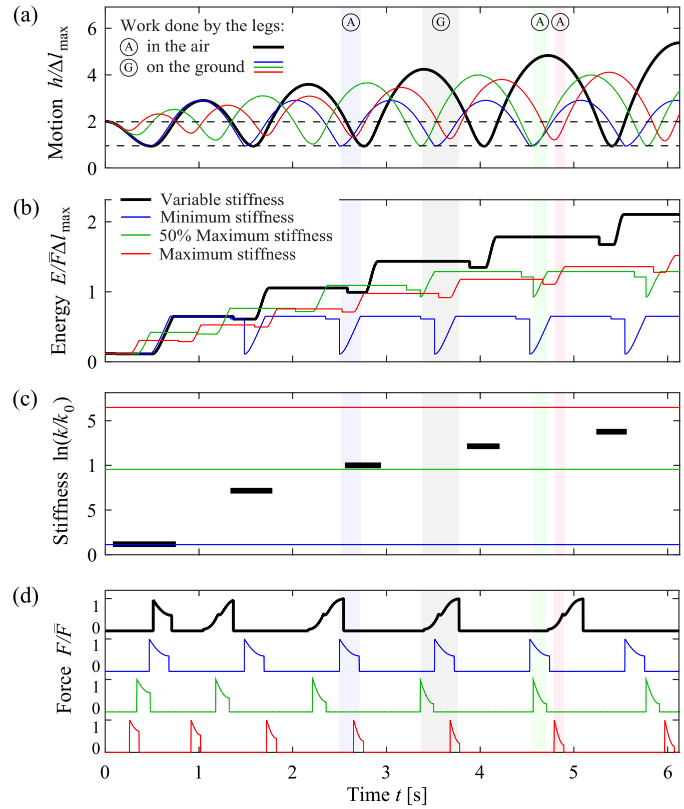


Fig. 6. The figure shows the (a) jump height, (b) energy, (c) stiffness, and (d) leg force versus time, given the velocity-dependent leg force, the mass of an ideal exoskeleton, and the time-delay in applying force on the ground, as defined in Sections III-A-III-C.

amount of work the human can do on the ground, see the reduced energy increments per jump in Fig. 6(b) (red and green lines compared to the black line). In the proposed augmentation method, the fixed stiffness spring allows the human to do work in the air, see Fig. 1a(A). Because the aerial phase of motion is much longer than the ground contact phase, humans can move their legs slowly in the air. Consequently, the work done becomes largely independent of the force-velocity limit of the limb, see the increments in energy per jump in Fig. 6(b) (black line).

### B. Time delay

In the proposed augmentation method, the human preloads a fixed stiffness spring in the air and the energy stored in the preloaded spring is released by a clutch mechanism when the legs are maximally bent, see Fig. 1a(B-C). Unlocking the spring with a clutch mechanism may introduce a small but nonzero time delay $\Delta t > 0$. Therefore, it is of interest to investigate the effect of time delay on the proposed augmentation method.

Due to a small mismatch between the actual and the exact time when the spring is unlocked $(k_\infty/k_0)(g\Delta t^2/\Delta l_{\max}) \ll 1$, the work done by the human using the variable stiffness exoskeleton $k_n = k(h_n) \in [k_0, k_\infty]$ is given by:

$$\frac{W(\Delta t)}{W(0)} \approx 1 - \left(2\frac{k_n}{k_0} - 1\right)\frac{g\Delta t^2}{\Delta l_{\max}} \tag{20}$$

while the saturation jump height is given by:

$$\frac{h_\infty(\Delta t)}{h_\infty(0)} = \lim_{n\to\infty} \frac{W(\Delta t)}{W(0)} \approx 1 - \left(2\frac{k_\infty}{k_0} - 1\right)\frac{g\Delta t^2}{\Delta l_{\max}}. \quad (21)$$

These expressions show that for any mismatch in unlocking the preloaded spring $\Delta t \neq 0$, both the work done by the spring and the saturation jump height will be reduced. In particular, unlocking the preloaded spring with a time delay of a typical mechanical clutch $\Delta t = 25$ ms [47] will result in one quarter of the ideal maximum jump height, see Fig. 5 compared to Fig. 3. This detrimental effect is mitigated by choosing the minimum stiffness $k_n = k(h_n)$ (2) as shown by (20) and (21). Therefore, the added benefit of the proposed variable stiffness augmentation method is that it minimizes the effect of time delay in every jump.

Although a mismatch in the exact timing to apply force by the legs may also exist, the effect of a velocity-dependent leg force (Section III-A) is dominant in our analysis. In particular, despite the assumed perfect timing $\Delta t = 0$, the limited force due to the increased velocity of the leg led to an order of magnitude lower saturation jump height when using the fixed stiffness augmentation method, see Fig. 5 compared to Fig. 3.

### C. Mass of the Exoskeleton

The variable stiffness spring is required to store a considerable amount of energy at saturation ($E_\infty \approx 3600$ J, $h_\infty \approx 4.8$ m), therefore the mass of the spring $m_{\text{spring}}$ may not be negligible. Although the contact between the exoskeleton and the ground is not completely rigid, here we assume the worst-case scenario in which the mass of the spring adds to the mass of the foot and thus directly increases the collisional energy loss at touchdown:

$$\Delta E_{\text{collision}} = (m + m_{\text{spring}})gh_n. \quad (22)$$

Given this collisional energy loss, the minimum required energy density of the spring to reach the saturation height is:

$$\frac{E_\infty}{m_{\text{spring}}} = \frac{W}{m_{\text{spring}}}\left(1 + \frac{M}{m + m_{\text{spring}}}\right). \quad (23)$$

Assuming that (i) the spring is made of fiberglass-reinforced polymer, which possesses a flexural energy density of $E_\infty/m_{\text{spring}} \approx 2000$ J/kg [52], (ii) the average force of the leg is velocity dependent (19), (iii) there is a switching time delay of the clutch mechanism (25 ms), the mass of the spring that can store all the energy required to reach the saturation height of $h_\infty = 4.8$ m in Fig. 5 is $m_{\text{spring}} = 1.9$ kg. The exoskeleton can be heavier than this spring, bu the collisional energy loss at touchdown should not exceed the kinetic energy of the foot and this $m_{\text{spring}} = 1.9$ kg ideal spring exoskeleton.

### D. Energy Cost

Variable stiffness springs are not passive devices; energy is required to change stiffness using an actuator [49], [50], [53], [54]. However, if the variable stiffness spring *does no output work*, the energy cost to change stiffness can be made negligible compared to the energy loss when using a fixed stiffness exoskeleton, see Section II-C.

Regardless of the practical realization of the variable stiffness spring, the energy cost per unit time to change stiffness can be decomposed into two parts:

$$\dot{E}_{\text{VS}} = \dot{E}_{\text{I}} + \dot{E}_{\text{II}} \quad (24)$$

where $\dot{E}_{\text{I}}$ is the rate of change of the potential energy stored by the spring while $\dot{E}_{\text{II}}$ is the power required for stiffness modulation when the spring stores no potential energy. Using this decomposition, we will subsequently show how to reduce the energy cost of stiffness modulation for the two components in (24).

(i) The rate of change in potential energy is given by

$$\dot{E}_{\text{I}} = \frac{1}{2}\dot{k}x^2 + kx\dot{x} \quad (25)$$

where $\dot{E}_{\text{I}}$ is zero if the stiffness $k$ is changed when no potential energy is stored in the spring

$$x = 0 \Rightarrow \forall k : \dot{E}_{\text{I}} = 0. \quad (26)$$

Under this condition, neither increasing nor decreasing stiffness will do mechanical work. Therefore, in order to achieve low energy cost human augmentation, we promote stiffness modulation without doing mechanical work. In the considered hopping task, this is achieved by changing stiffness when the legs are straight, i.e in the aerial phase of jumping (see Figs. 1a(A), 2 and 6). In more general terms, stiffness should be changed when no potential energy is stored in the spring.

(ii) The power required for stiffness modulation when no potential energy is stored in the device is given by

$$\dot{E}_{\text{II}} = p_1(k, \dot{k}) + p_0(k) \quad (27)$$

where the first term depends on the speed of modulation (faster modulation requires more power) while the second term represents the energy dissipation (for example due to Coulomb friction and Joule heating in electromechanical actuators).

Changing stiffness in a large range does not imply large energy cost because it is possible to design a variable stiffness spring where the baseline power $p_0$ is independent of the stiffness $k$ [55]; see for example [50], [54] where

$$\forall k : \frac{\partial p_0}{\partial k} = 0 \text{ and } p_0 \approx 0. \quad (28)$$

However, changing stiffness rapidly may result in large energy cost, because:

$$\dot{k} \to \infty \Rightarrow p_1(\dot{k}, k) \to \infty. \quad (29)$$

Consequently, to minimize the energy cost of stiffness modulation the mechanical design of the variable stiffness spring should possess a *baseline power independent of stiffness* (28) while stiffness should be changed *as slow as possible (29).* However, the speed of stiffness modulation is generally task dependent. Indeed, for a given task, the speed of stiffness modulation is defined by the time allowed to change stiffness and the required stiffness change. In the considered hopping task, the stiffness change $\Delta k_n$ is given by:

$$\frac{\Delta k_n}{k_0} = \frac{m}{M + m}\frac{h_\infty - h_n}{\Delta l_{\max}} \quad (30)$$

while the time to change the stiffness, i.e. the time spent in the air between jumps is:

$$\Delta t_{n\text{air}} = \sqrt{\frac{8h_n}{g}}. \tag{31}$$

As the number of jumps increases, the stiffness change tends to zero and the time between jumps monotonically approaches its largest value:

$$\Delta k_\infty \to 0 \;\text{ and }\; \Delta t_{\infty\text{air}} \to \sqrt{\frac{8h_\infty}{g}}. \tag{32}$$

Therefore, the fastest stiffness change is required during the first jump $n = 1$, see Figs. 2 and 6.

For the human data in Table I, the stiffness increment in the first jump is $\Delta k_1 \approx 2630$ Nm$^{-1}$ and the alloted time to change stiffness is $\Delta t_{1\text{air}} \approx 0.643$ s which give the largest stiffness change of $\dot{k}_{\max} = \Delta k_1/\Delta t_{1\text{air}} \approx 4090$ Nm$^{-1}$s$^{-1}$. This requirement, in combination with the energy storing capacity of $E_\infty \approx 3600$ J to reach the saturation height of $h = 4.8$ m, are beyond the capability of current variable stiffness spring designs [50].

### E. Modeling Assumptions

In the previous sections we (i) considered the motion of the center of mass of the human and (ii) assumed that the exoskeleton uses a linear variable stiffness spring. Here we show that these assumptions do not restrict the results presented in this paper.

(i) *Model of the human:* The human body has more than two hundred degrees of freedom and is actuated by more than six hundred muscles [51]. During hopping motion, the net effect of the muscles is to move the center of mass vertically while maintaining balance. The motion of the center of mass thus becomes a one degree of freedom movement which can only be influenced by external forces. The two main external forces are the gravitational and vertical ground reaction force. Any internal forces in the body are canceled out, and the dynamics of a complex human-variable stiffness exoskeleton can be described by the variable stiffness spring acting on the center of mass of the human. Because the internal degrees of freedom and the internal muscle forces do not affect the motion of the center of mass, a more comprehensive human model does not change the obtained results or provide additional insight regarding the motion of the center of mass of the augmented human. However, a comprehensive model, similar to [1], [44], can be used to study the effect of the exoskeleton on the body, which is beyond the scope of this paper.

(ii) *Model of the exoskeleton:* To facilitate the presentation and interpretation of the results, we have assumed a linear variable stiffness spring, though this assumption is not essential to implement the proposed augmentation method. The only essential requirement is that the exoskeleton is capable of storing a variable amount of energy within the deformation limit of the human limb. For a linear variable stiffness exoskeleton, the energy stored is given by:

$$E_{\text{exo}} = \frac{1}{2}k\Delta l_{\max}^2 = \frac{M}{M+m}E_n. \tag{33}$$

For a nonlinear exoskeleton, the energy stored is given by:

$$E_{\text{exo}} = \int_0^{\Delta l_{\max}} F_{\text{exo}}(k,l)\mathrm{d}l = \frac{M}{M+m}E_n \tag{34}$$

where $F_{\text{exo}}(k,l)$ is the force-deflection characteristic of the variable stiffness spring. Therefore, regardless of the force-deflection curve $F_{\text{exo}}(k,l)$, a nonlinear variable stiffness spring may be used to accumulate energy similar to a linear one. In fact, any device that can be actively controlled to store varying amounts of energy can be used to augment the human similar to a linear variable stiffness exoskeleton.

Finally, we note that there may be nonlinear variable stiffness spring designs $F_{\text{exo}} = F_{\text{exo}}(k,l)$ that could improve human comfort over the linear spring $F_{\text{exo}} = kl$ used in this paper. However the relation between human comfort and the exoskeleton design has yet to be experimentally investigated.

## IV. Discussion and Conclusion

An augmentation method has been proposed to increase mobility by increasing the maximum kinetic energy the limbs can transmit to the human. Examples of tasks that benefit from increased kinetic energy are high jumping, horizontal leaping, and fast running. Here we discuss prospective applications of the proposed augmentation method and recall prior experimental results that corroborate the findings of this paper.

Human hopping and running motion has been modeled by a linear spring-mass system where stiffer legs provide greater forces for the same deflection of the legs [30], [42]. Consistent with human experiments, the minimalistic spring-mass model suggests that lower leg stiffness allows the legs to deflect more, but results in no aerial rebound [32], while greater leg stiffness allows faster rebound, but decreases the deflection of the leg for the same initial drop height [33]. These findings suggest that there is an optimum leg stiffness to store some of the kinetic energy of the center of mass, while allowing larger deflection of the leg to increase the work provided by the human [56].

However, human legs possess insufficient elasticity to allow energy accumulation over multiple jumps. For example, in the experimental study of drop jumps from different heights [43], the authors reported no increase in jump height despite the fact that the increased drop heights provided more gravitational potential energy that could have been recovered using the energy storing capability of the leg, and reused in the subsequent jump. However, the collisional energy loss at ground contact also increases with drop height, and this loss can saturate the high of the jump when the sum of the energy stored in the leg and the work done in the next jump is equal to or less than the collisional energy loss. Consequently, the energy storing capacity of the limb is negligible compared to what is required to accumulate energy through multiple jumps.

Human augmentation with fixed stiffness parallel leg elasticity for walking and hopping has been long proposed [57], and more recently investigated [21], [22]. In the former paper the authors show a reduced energy cost of walking when using a fixed stiffness exoskeleton compared to an exoskeleton that has no energy storing capacity. In the latter paper, the authors designed an exoskeleton to store energy within the maximum contraction of the legs. This has been shown to appreciably reduce the metabolic cost of hopping up to $30\%$ in a single jump. Using a variable stiffness exoskeleton we theorize a method to increase leg output work beyond the energy of a single jump. We predict that the quasi-passive variable stiffness augmentation method proposed in this paper could provide substantial improvement over a fixed stiffness exoskeleton (Section II), despite not contributing any net mechanical work.

Fixed stiffness parallel ankle elasticity has recently been shown to decrease the metabolic cost of walking by up to $7.2\%$ [23]. The authors found an optimal stiffness over nine subjects to minimize metabolic energy cost. The reason why fixed parallel stiffness is sufficient for this case of walking augmentation is due to the fixed walking speed, in which the spring could be optimized to provide part of the ankle torque at that specific speed. We posit that walking at different speeds necessitates a variable stiffness ankle exoskeleton to quasi-passively exert the ankle torques that differ at different speeds. Furthermore, we note that it is not the physical stiffness but the dimensionless stiffness [58] that can be assumed constant across subjects, because humans with different height and weight may prefer different optimal stiffness values even when walking at the same speed [42].

Prior works show that the top running speed of humans and animals is limited by the force required to recover and redirect the vertical momentum lost in each step [59]. As forward speed increases, the legs exert less force due to less time available to generate force. By adding a spring in parallel with the leg, the vertical momentum of the body may be recovered and passively redirected upwards. Some energy input is required to replace the collisional energy loss of the feet, but far less than what is required when losing the vertical momentum of the entire body. However, the stiffness of the exoskeleton should be chosen to redirect vertical momentum within the ground contact time, which is a function of the forward speed [59]. A variable stiffness could be used to implement resonant matching at different speeds for running at constant speed and accelerating to greater speeds [60].

The proposed quasi-passive variable stiffness augmentation can be used by a human to accumulate more kinetic energy, and therefore move faster and further. However, increased kinetic energy may also *enable the human to move in unexpected ways*. For example, hopping in place is an energy intensive activity (often avoided by un-augmented humans) in which kinetic energy cannot be accumulated in each step like in running. However, when a person is augmented with a variable stiffness exoskeleton, hopping in place provides a “vertical runway”; it can be used to accumulate energy to leap across a great distance, for example, across a collapsed floor of a building during a fire rescue, where limited space prevents accumulation of kinetic energy by running.

In this paper, we assumed that the exoskeleton is capable of absorbing the kinetic energy of the body upon landing and transferring its stored elastic potential energy to the body during takeoff. In practice, efficient energy-transfer between a wearable exoskeleton and a human is a significant engineering challenge [61] because the human-exoskeleton interface introduces energy dissipation. Furthermore, humans have a maximum impact force tolerance; for example, the maximum drop height that can be endured by an un-augmented human without injury when landing feet-first is approximately three meters [62]. However, the survivability and injury rate of falls is highly dependent on the deflection of the leg used to decelerate the motion, which could be maximized by the augmented human using the variable stiffness exoskeleton. Consequently, although the free-fall impact data from un-augmented humans provides a useful safety guideline, it is different from the augmented case where the force to decelerate is exerted not only by the leg but also by the exoskeleton. Experiments, beyond the scope of this theoretical study, may be required to better understand the safety limits of the proposed variable stiffness augmentation method.

One of the main factors preventing mainstream adoption of exoskeleton devices is power consumption [63], because an exoskeleton that does mechanical work requires large motors and heavy batteries even when using state-of-the-art technologies. In this paper we have theoretically shown that *doing work is not fundamental to providing significant mechanical advantage to humans.* In fact, it is possible to go considerably beyond biological limitations using a quasi-passive variable stiffness exoskeleton that does no mechanical work. The only energy cost of such device is to change stiffness without doing work, which is independent of movement and may be effectively minimized as shown in Section III-D, see [50], [55].

Although the development of lightweight, wearable, and energetically autonomous quasi-passive variable stiffness exoskeletons remains an engineering challenge [54], [64], such devices may enable mainstream adoption of mobility enhancing exoskeletons.

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