Name: Solutions

The objective of this exercise is to plot the root locus corresponding to a unity feedback system with a characteristic polynomial given as follows:

$$1 + KG(s)H(s) = 1 + K\frac{(s+2)}{s^2+2s+2}.$$

Answer the following questions. Show your work.

1. Determine the poles and zeros of the open loop transfer function ($G(s)H(s)$). How many branches does the RL have?

$z = -2 \Rightarrow n_z = 1$ Number of branches $= n_p = 2$

$P_{1,2} = -1 \pm j \Rightarrow n_p = 2$

2. Determine the asymptote angles ϕ_q as well as their point of intersection σ_A. Recall that:

$$\phi_q = \frac{(1+2q)180}{n_p - n_z} \text{ deg}, \forall q = 0, 1, 2, \ldots, n_p - n_z - 1$$

$$\sigma_A = \frac{\sum_{i=1}^{n_p} Re(p_i) - \sum_{j=1}^{n_z} Re(z_j)}{n_p - n_z} = \frac{-1-1-(-2)}{2-1} = 0$$

$\forall q = 0, 1, 2 \cdots, \underbrace{n_p - n_z - 1}_{2-1-1} = 0$

$$\phi_q = \frac{(1+2q)180}{n_p - n_z} = \frac{(1+2\cdot 0)180}{2-1} = 180^\circ$$

3. Determine any breakaway/break-in points. If none exist, state none!

Rule 8-1

$\boxed{1+K\,G(s)\,H(s)=0} \Rightarrow 1+K\dfrac{(s+2)}{s^2+2s+2}=0$

$$K=-\frac{s^2+2s+2}{s+2}$$

$\Downarrow$

$$\frac{dK}{ds}=-\frac{d}{ds}\left(\frac{s^2+2s+2}{s+2}\right)$$

$$0=-\frac{(2s+2)(s+2)-(s^2+2s+2)}{(s+2)^2}$$

$$0=2s^2+4s+\cancel{2s}+4-s^2-\cancel{2s}-2$$

$$s^2+4s+2=0$$

$$s^*_{1,2}=-2\pm\sqrt{2}$$

Rule 8-2

$K(s^*)=-\dfrac{1}{G(s^*)H(s^*)}=K^*$ is real positive $(K^*>0)$

$K(s^*)=-\dfrac{s^2+2s+2}{s+2} \xrightarrow{\text{when } s^*=-2+\sqrt{2}} K=-\dfrac{\cancel{4}-4\sqrt{2}+2+2\sqrt{2}-\cancel{4}+2}{\cancel{-2}+\sqrt{2}\cancel{+2}}$

$\xrightarrow{s^*=-2-\sqrt{2}}$ $=-\dfrac{4-2\sqrt{2}}{\sqrt{2}}=-(2\sqrt{2}-2)$

$K>0$ ✓ $=-\cdot+=-$ X

breakaway Point $K<0$

$s_2=-2-\sqrt{2}$

$\boxed{\text{Rule-8.3}}$

$$K = -\frac{s^2+2s+2}{s+2} \Rightarrow K(s) = -\frac{s^2+2s+2}{s+2}$$

$$K'(s) = -\frac{(2s+2)(s+2)-(s^2+2s+2)}{(s+2)^2} = -\frac{2s^2+4s+2s+4-s^2-2s-2}{s^2+4s+4}$$

$$K'(s) = -\frac{s^2+4s+2}{s^2+4s+4}$$

$$K''(s) = \left(K'(s)\right)' = -\frac{(2s+4)(s^2+4s+4)-(2s+4)(s^2+4s+2)}{(s^2+4s+4)^2}$$

$$= -\frac{(2s+4)(\cancel{s^2}+\cancel{4s}+4-\cancel{s^2}-\cancel{4s}-2)}{(s^2+4s+4)^2} = -\frac{4s+8}{(s^2+4s+4)^2}$$

$$s^* \longrightarrow s \Rightarrow K''(s_1^*) = -\frac{4(-2+\sqrt{2})+8}{(\quad)^2} = -\frac{4\sqrt{2}}{\text{"}+\text{"}} < 0$$

$s_1^* = -2+\sqrt{2}$

$s_2^* = -2-\sqrt{2}$

$$K''(s_2^*) = -\frac{4(-2-\sqrt{2})+8}{(\quad)^2} = -\frac{-4\sqrt{2}}{\text{"}+\text{"}} > 0$$

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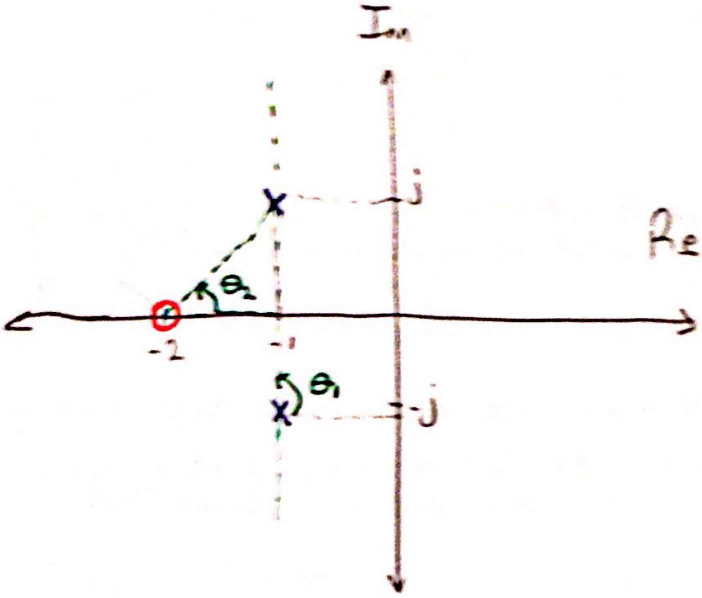
4. Determine the angle of departure/arrival, if any. Recall that:

$$\text{AoD from a complex pole: } \boxed{\varphi_p = 180 - \sum_i \angle p_i + \sum_j \angle z_j}$$

$$\text{AoA at a complex zero: } \varphi_z = 180 + \sum_i \angle p_i - \sum_j \angle z_j$$

No complex zero

So no AoA



$\Phi_{p_1} = 180 - \theta_1 + \theta_2$

$= 180 - 90 + \tan^{-1}\left(\frac{1}{1}\right)$

$= 180 - 90 + 45 = 135^\circ$

$\Phi_{p_2} = -\Phi_{p_1} = -135^\circ$

5. Sketch the root locus—no need to find the crossings with the $j\omega$ axis.

O → zeros

X → Poles