Name:

1. You are given this system:

$$x(k+1) = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 2 \end{bmatrix} u(k), \quad u(k) = 0.5^k, \quad x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

Find x(n). You might find this equation to be useful:

$$x(k) = A^{k}x(0) + \sum_{j=0}^{k-1} A^{k-1-j}Bu(j) = A^{k}x(0) + \sum_{j=0}^{k-1} A^{j}Bu(k-1-j)$$

2. Consider this dynamical system

$$x(k+1) = A(k)x(k) + B(k)u(k), y(k) = C(k)x(k) + Du(k).$$

Note that *A*, *B*, *C* are all **time-varying**. Given that you have four sets of inputoutput data:

$$(y(0), u(0)), (y(1), u(1)), (y(2), u(3)), (y(3), u(3))$$

and x(0) is unknown, derive an equation that would allow you to obtain x(0) as follows:

 $\bar{A}x(0) = \bar{b}$ 

where  $\bar{A}$  and  $\bar{b}$  are quantities that you should determine, then discuss the conditions for  $\bar{A}$  that allows you to generate a unique x(0). Note that in this problem, the number of states is much larger than the output measurements.