| THE UNIVERSITY OF TEXAS AT SAN ANTONIO | QUIZ \# 6 |
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| EE 5143 | Ahmad F. Taha |
| LINEAR SYSTEMS AND CONTROL | October 10, 2017 |

Name:

1. You are given this system:

$$
x(k+1)=\left[\begin{array}{cc}
0.5 & 0 \\
0 & 2
\end{array}\right] x(k)+\left[\begin{array}{l}
2 \\
2
\end{array}\right] u(k), \quad u(k)=0.5^{k}, \quad x(0)=\left[\begin{array}{c}
1 \\
-1
\end{array}\right] .
$$

Find $x(n)$. You might find this equation to be useful:

$$
x(k)=A^{k} x(0)+\sum_{j=0}^{k-1} A^{k-1-j} B u(j)=A^{k} x(0)+\sum_{j=0}^{k-1} A^{j} B u(k-1-j)
$$

2. Consider this dynamical system

$$
x(k+1)=A(k) x(k)+B(k) u(k), y(k)=C(k) x(k)+D u(k) .
$$

Note that $A, B, C$ are all time-varying. Given that you have four sets of inputoutput data:

$$
(y(0), u(0)),(y(1), u(1)),(y(2), u(3)),(y(3), u(3))
$$

and $x(0)$ is unknown, derive an equation that would allow you to obtain $x(0)$ as follows:

$$
\bar{A} x(0)=\bar{b}
$$

where $\bar{A}$ and $\bar{b}$ are quantities that you should determine, then discuss the conditions for $\bar{A}$ that allows you to generate a unique $x(0)$. Note that in this problem, the number of states is much larger than the output measurements.

